Aging, Social Security Reform and Factor Price in a Transition Economy*

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Abstract

This paper explores the macroeconomic and welfare implications of aging and social security reform in Japan. Based on the overlapping generations model with idiosyncratic income risk, we demonstrate that aging induces capital deepening and wage increase in the transition path. We consider four social security reform plans; (1) the reduction of the replacement rate by half, (2) full privatization, (3) capital income tax, and (4) consumption tax. We compute the transition paths of each case, and find that the introduction of capital income tax improves the welfare of the young and future households based on the cohorts' viewpoint. On the contrary, the introduction of consumption tax does not improves the social welfare of the economy because of intragenerational inequality and substitution effect. Further, we show that under the transition, the earnings profiles become a more hump-shaped and consumption profiles become steep with the changes in the replacement rate.

Keywords: Aging, Factor Price, Heterogeneity, Social Security Reform, Transition

JEL Classification: H55, D31, D33, E24, E25.

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1 Introduction

As the United Nations (2007) reports, Japan is one of the most aging societies in the world. The reduction in the size of the of working population and the rise of old-age dependency rate causes serious problems in social security reform. As far this issue is concerned, in order to sustain the social security system, some reform should be considered. However, almost all social security reforms may induce well-known intergenerational conflicts. Moreover, the allocation of wealth and consumption are rather complicated under the transition to an aging population. Therefore, we need to give careful consideration to the macroeconomic and welfare implications of this transition.

In aging society, there are two possible scenarios. First, the capital-labor ratio decreases because a large burden of social security tax induces lower savings. Moreover, an increase in the number of retired households implies that many households decumulate their wealth for lifecycle reason. Second, in contrast, the capital-labor ratio may increase when labor supply measured in terms of efficiency decreases, because the social security tax burden creates a distortion on labor supply decision. Based on the usual Cobb=Douglas-type production function, the capital-labor ratio determines factor prices, the interest rate, and wage level. Our main focus in this paper is the dynamics of allocations under the transition to an aging society, with a particular focus on the dynamics of factor prices and the welfare of each cohort. As Davila et al. (2006) reveals, factor prices have different effects on the wealth rich and the poor. Moreover, the changes in factor prices have different effects on employees and retirees. Therefore, to consider social security reform and evaluate the welfare of households, we need to investigate the dynamics of macroeconomic statistics and factor prices as well as the social security tax rate.

We consider the macroeconomic and welfare implications of aging and social security reform under the transition by using the overlapping generations model with idiosyncratic income risks. This model features not only intergenerational heterogeneity but also intragenerational heterogeneity because of the realization of idiosyncratic income shocks. Thus, we consider the decisions of different asset and income level for each age group. Our finding indicate that further capital deepening can be expected in the near future in Japan. Further, if the capital-labor ratio increases, then based on the Cobb=Douglas-type production function, the equilibrium wage increases by 6% and the interest rate decreases by 1.5% in the transition path. Therefore, younger households have a chance to obtain greater labor earnings. However, the output per capita decreases by 20% because of the decrease in the aggregate capital and labor supply. If some type of social security reform is implemented, such as the reduction of the replacement rate, there will be further capital deepening. With the strong incentive toward savings for retirement, households supply more labor and accumulate wealth. This may increase the per capita GDP and welfare due to a significant reduction of the replacement rate. Not surprisingly, these reforms induce a strong intergenerational conflict measured in terms of the cohorts' welfare. In addition, we investigate the other sources of finance for the social security system, namely consumption tax and capital income tax. We find that the introduction of capital income tax flattens the factor prices path over time, and capital income tax weakly improves the welfare of the young and future generations. On the contrary, the consumption tax induces that young households consume more, and intragenerational consumption inequality of young generations increases. Therefore, at least in our estimation, the consumption tax does not improves the welfare of the economy. With both of these new financing schemes, the payroll tax rate on labor income decreases by 2% $\sim$ 4%.

Our paper is a branch of the traditional research that analyses transition dynamics of aging and social security reform based on the overlapping generations model. Since the initial research of Auerbach and Kotlikoff (1987), there have been many researches on demographic structure and social security including Huang et al. (1997) and De Nardi et al. (1999). Huggett (1996) and İmrohoroğlu et al. (1995) extend these papers to consider not only intergenerational but also intragenerational heterogeneity by including the element of idiosyncratic income uncertainty. Huggett (1996) find that the model with intragenerational heterogeneity generates a close-to-real wealth inequality measured by the Gini coefficient. İmrohoroğlu et al. (1995) demonstrates that the optimal replacement is zero when the discount rate is positive. Our approach is very similar to those of Conesa and Krueger (1999) and Nishiyama and Smetters (2005b). Although these works investigate the transition path of an aging economy with/without the social security reform of replacement rate reduction, they do not consider financing reform such as consumption tax and capital income tax. Therefore, we focus on the transition dynamics with such social security reform in Japan.

The paper is organized as follows. In Section 2, we present our basic model and present some scenarios to be considered. Section 3 introduces the calibration parameters for the Japanese economy. In Section 4, we report our main results on the transition path. Section 5 discusses the implications of factor prices and the behavior of households on the transition analysis. Section 6 presents the robustness of our analysis and Section 7 provides the concluding remarks.

2 The Model

2.1 Demographic Structure

We consider the overlapping generations model with a continuum of households.[1] In the model, time is discrete. The lifespan of the households is a maximum of J-years, but they face mortality risks. The number of households aged $j \in (0, \dots, 20, \dots, J)$ in period t is denoted by $\mu_{j,t}$. A fraction of households $(1-\phi_{j,t})$ exits the economy owing to death, and $\mu_{j+1,t+1} = \phi_{j,t}\mu_{j,t}$ is the population of households aged $j+1$ at period $t+1$, where $\phi_{j,t}$ is

[1] Our model describes population dynamics and total factor productivity growth. Thus, we need to distinguish between nominal and detrended variables to solve the model. For details, see the Appendix.

the survival probability. We assume that households begin economic activity at $j = 20$. Because households are in their childhood at $j = 0, 1, \ldots, 19$, they do not engage in consumption or employment but are included in the population dynamics for computing the future fertility rate. By assumption, $\mu_{J+1,t} = \phi_{J,t} = 0$. Let $\mu_t = (\mu_{1,t}, \ldots, \mu_{J,t})$ denote the population distribution in period t. Therefore, the population dynamics in our economy is expressed in the following matrix form:

$$\mu_{t+1} = \begin{bmatrix} 1+\psi_t & 0 & 0 & \cdots & 0 \\ \phi_{1,t} & 0 & 0 & \cdots & 0 \\ 0 & \phi_{2,t} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \phi_{J-1,t} & 0 \end{bmatrix} \mu_t \equiv \Gamma_t \mu_t,$$

where ψ_t is the population growth rate of age 0 from t to $t+1$.[2] New households enter the economy in period $t+1$ as $\mu_{0,t+1} = (1+\psi_t)\mu_{0,t}$, where the population growth rate is derived from the net fertility rate. The aggregate population including children at t is $N_t = \sum_{j=0}^{J} \mu_{j,t}$. We denote the population growth rate from period t to $t+1$ as n_t, i.e., $N_{t+1} = (1+n_t)N_t$. Further, we assume that the aggregate population in period 0 is normalized to one, and the population growth rate is constant in the stationary state (i.e., $\sum_{j=0}^{J} \mu_{j,0} = N_0 = 1$, $N_t = (1+n)^t N_0$). Although the population distribution is constant over time at the stationary state (i.e., $\mu_{j+1,t+1}/N_{t+1} = \mu_{j,t}/N_t$), the population distribution varies under the transition path. In the following section, we consider both the stationary and transition economy.

2.2 Households

2.2.1 Objective Function

A household born in period t has a lifespan of at most J periods, elastically supplies labor until age j_r, and faces idiosyncratic uncertainty with regard to its individual labor productivity. The objective function of the household in period t is written as follows:

$$U_t = E_{20,t} \left\{ \sum_{j=20}^{J} \beta^{j-1} \left(\prod_{i=20}^{j-1} \phi_{i,t} \right) u(c_{j,t+j-20}, \bar{\ell} - \ell_{j,t+j-20}) \right\},$$

where $\beta > 0$ is a discount factor. All households have labor endowment $\bar{\ell}$ and supply their labor, $\ell_{j,t+j-20} \in [0, \bar{\bar{\ell}}]$, at j.

Since households of age $j \in \{20, \ldots, j_r\}$ are of employable age, they can elastically supply labor. Thereafter, i.e., $j \in \{j_r+1, \ldots, J\}$, the households retire and receive social security benefit from the government.

[2] See also Ríos-Rull (2001) for the details on the transition of population distribution.

2.2.2 Idiosyncratic Risk and Budget Constraint

All household face idiosyncratic income risk and also have deterministic labor productivity. The average earnings must be reflective of age-specific average labor productivity. The average labor productivity grows when households are young and peaks at middle-age around the 50s; in other words, the efficiency of a household has an hump-shaped across age groups. We denote the deterministic productivity measured by hourly wage as $\{\eta_j\}_{j=20}^{j_r}$.

Moreover, all households face idiosyncratic labor productivity risks when they are in employment. Following Storesletten, Telmer, and Yaron (2004), we assume that the idiosyncratic risk comprises three components; (1) *transitory shocks*, (2) *persistent shocks*, and (3) the *fixed effect*. The labor productivity process e_t is specified as follows.

$$\ln e_t = \alpha + z_t + \kappa_t, \tag{1}$$

$$z_t = \rho z_{t-1} + \varepsilon_t. \tag{2}$$

The fixed effect is denoted by the variance of α, and the transitory shock as that of κ_t. A persistent component of the idiosyncratic shock is represented by z_t, which is composed of the persistence parameter ρ and the shock ε_t. Let $s = (\alpha, z, \kappa)$ denotes a state of the idiosyncratic shocks for an individual household. We assume that all idiosyncratic shocks are discretized. The persistent component follows a Markov chain, and the transition probability is written as $\pi(z_{j+1}|z_j) \in (0, 1)$. We assume that the average efficiency profile and the stochastic productivity are independent of time t and across agents. Thus, the before-tax labor earning of each age group is determined by $y_{j,t} = w_t \eta_j e_j \ell_{j,t}$, where w_t is the economy-wide wage level.

The government grants social security benefits through a constant payroll tax from labor earnings, and retired households receive the social security benefit. The constant payroll tax rate is denoted as τ_t^{ss}. For a later analysis, we define the consumption tax rate and linear capital income tax rate as τ_t^c, and τ_t^a, respectively.[3] After retirement, a household receives social security benefit $w_t b(\tau_t, W_{g,t})$, where $b(\tau_t, W_{g,t})$ is a replacement rate determined from each tax rates $\tau_t \equiv (\tau_t^{ss}, \tau_t^c, \tau_t^a)$ and social security funds $W_{g,t}$.

A household has some asset holdings $a_{j,t} \in A \subseteq \mathbb{R}_+$ at age j and in period t. Since households face mortality risk, some household may die with positive assets. We assume that the existence of life insurance and that the accidental bequests are redistributed over living households by an insurance company, which implies that capital income that living households can receives increases by $1/\phi_{j,t}$.[4] Thus, the budget constraints for

[3] Conesa and Krueger (2005) consider an optimal progressive tax system on income flow $w_t \eta_j e_j \ell_{j,t} + r_t a_{j,t}$. For our purpose, we need to divide the income sources because we will focus on incentives on savings and consumption for each taxes.

[4] In previous version of this paper, we have assumed that the accidental bequests are collected by the government and redistributed for all households by lump sum manner. Those differences does not make large changes of macroeconomic variables in our analysis. Hansen and İmrohoroğlu (2006) reveals that a lack of annuity markes affects large impacts on lifecycle consumption profile.

employees and retirees are

$$\begin{array}{ll}(1+\tau_t^c)c_{j,t}+a_{j+1,t+1}\le(1-\tau_t^{ss})\,w_t\eta_j e_j\ell_{j,t}+(1+(1-\tau_t^a)r_t/\phi_{j,t})a_{j,t}, & :\text{employee}\\ (1+\tau_t^c)c_{j,t}+a_{j+1,t+1}\le w_t b(\tau_t^{ss},W_{g,t})+(1+(1-\tau_t^a)r_t/\phi_{j,t})a_{j,t}, & :\text{retiree}\end{array}$$

where r_t is the interest rate at t. We assume that households face a liquidity constraint, i.e., $a_{j,t}\ge 0$.

2.3 Behavior of Firms and the Factor Price

The aggregate production technology follows a Cobb=Douglas constant returns to scale production function

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta},$$

where A_t denotes the total factor productivity (TFP) in period t, K_t is an aggregate capital, and L_t is an aggregate labor supply measured by efficiency units. We assume that a sequence of the TFP is deterministic. Therefore, there are no aggregate uncertainties in our economy, and the aggregate productivity and the population growth is perfectly forecastable. We denote the gross growth rate of the TFP as $1+g_t=(\frac{A_{t+1}}{A_t})^{\frac{1}{1-\theta}}$.

The asset holdings and labor supply of each households differs even in the same cohort and age group due to idiosyncratic income uncertainty. We denote a fraction of households aged j with asset a and realized productivity s as $\Phi_t(a_j,s_j)$.[5] By construction, $\int d\Phi_t(a_j,e_j)=1$. The aggregate capital and labor supply are determined by the sums of each generation's capital and labor, as follows.

$$K_t=\sum_{j=20}^{J}\mu_{j,t}\int a_{j,t}d\Phi_t(a_j,e_j)+W_{g,t}, \tag{3}$$

$$L_t=\sum_{j=20}^{j_r}\mu_{j,t}\int \eta_j e_j\ell_{j,t}d\Phi_t(a_j,e_j). \tag{4}$$

The interest rate r_t and wage w_t at period t are determined as follows.

$$r_t=\theta A_t\left(\frac{K_t}{L_t}\right)^{\theta-1}-\delta,\ \ w_t=(1-\theta)A_t\left(\frac{K_t}{L_t}\right)^{\theta},$$

where δ is the depreciation rate.

2.4 The Government and the Social Security System

We assume that the government collects tax to finance social security benefits and redistributes it to retired households in a lump-sum manner, and we do not consider other government expenditures. In our setup, the social security system is basically governed

[5]For details of the distribution function, see the Appendix.

by the *Pay-as-you-go* system although the social security system in Japan is conducted as a "modified funded system". Accordingly, the government collects payments from employees and retirees using $(\tau_t^{ss}, \tau_t^c, \tau_t^a)$, and grants social security benefits through $b(\tau_t, W_{g,t})$.[6] We assume that the replacement rate is exogenously fixed and the corresponding tax rates are determiend endogenously. Because the old-age dependency rate has been considerably lower in the past years, the government holds positive social security trust funds in Japan. We denote the social security payments by the payroll tax as T_t^{SS}, by the consumption tax as T_t^C, by the capital income tax as T_t^A, and aggregate social security benefit as B_t. Therefore, if the contribution is greater than the benefit, the government increases its fund, and vice verse.

The government should satisfy the following budget constraints.[7]

$$
\begin{aligned}
W_{g,t+1} &= (1+r_t)W_{g,t} + (T_t^{SS} + T_t^C + T_t^A) - B_t, \qquad (5)\\
T_t^{SS} &= \sum_{j=20}^{j_r} \mu_j \int \tau_t^{ss} w_t \eta_j e_j \ell_{j,t} d\Phi_t(a_j, e_j) = w_t \tau_t^{ss} L_t,\\
T_t^C &= \sum_{j=20}^{J} \mu_j \int \tau_t^c c_{j,t} d\Phi_t(a_j, e_j) = \tau_t^c C_t,\\
T_t^A &= \sum_{j=20}^{J} \mu_j \int \tau_t^a a_{j,t} d\Phi_t(a_j, e_j) = w_t \tau_t^a K_t,\\
B_t &= \sum_{j=j_r+1}^{J} \mu_{j,t} w_t b(\tau_t, W_{g,t}) L_t = w_t b(\tau_t, W_{g,t}) L_t N_t^{\text{ret}},
\end{aligned}
$$

where N_t^{ret} is a fraction of retired households over the total population who have already entered the economy. We assume that the social security funds are invested into production and that they increase with the rise in the interest rate. Note that the average labor income of all workers is $w_t L_t$. Because the replacement rate $b(\tau_t, W_{g,t})$ is determined by a constant of the average labor income, $w_t b(\tau_t, W_{g,t}) L_t$ is the social security benefit that is exogenously determined by the government.

From the above, the aggregate states of the economy at period t is summarized as $X_t = (K_t, L_t, A_t, \Phi_t, \mu_t)$. The government's policy and funds are denoted as $\Omega_t =$

[6]In our model, we assume that the social security benefits are the same for all retirees and with the same fixed effect α. We do not consider a too complicated social security system but a realistic one because of the computational burden. For the redistributional effects of the social security system in the US, see Huggett and Ventura (1999) and Storesletten et al. (1998).

[7]At stationary state, output Y_t also grows by the rate of population growth $(1+n_t)$ and the TFP growth $(1+g_t)$. Thus, the government fund $W_{g,t}$ has positive trend of $(1+n_t)(1+g_t)$ in the stationary state. Normalized government budget constraints are determined by dividing $A_t^{\frac{1}{1-\alpha}} N_t$ both sides of equations.

$$\tilde{W}_{g,t+1} = \frac{1}{(1+n_t)(1+g_t)}[(1+r_t)\tilde{W}_{g,t} + (T_t^{SS} + T_t^C + T_t^A) - B_t]$$

Note that the wage, the aggregate consumption and the aggregate capital grow by $(1+n_t)(1+g_t)$.

$(\tau_t, W_{g,t}, b(\tau_t, W_{g,t}))$. Then, the Bellman equation of age j in period t is as follows.

$$v_t(a_j, s_j; X_t, \Omega_t) = \max_{c,a'} \left\{ u(c_{j,t}, \bar{\ell} - \ell_{j,t}) + \phi_{j,t}\beta E v_{t+1}(a', s'; X_{t+1}, \Omega_{t+1}) \right\}, \quad (6)$$

subject to

$$(1+\tau_t^c)c_{j,t} + a_{j+1,t+1} \leq (1-\tau_t^{ss})\, w_t \eta_j e_j \ell_{j,t} + (1+(1-\tau_t^a)r_t/\phi_{j,t})a_{j,t}, \quad (7)$$

$$(1+\tau_t^c)c_{j,t} + a_{j+1,t+1} \leq w_t b(\tau_t, W_{g,t}) + (1+(1-\tau_t^a)r_t/\phi_{i,t})a_{j,t}. \quad (8)$$

The intertemporal and intratemporal first order conditions are such that[8]

$$u_c(c_{j,t}, \bar{\ell} - \ell_{j,t}) \geq \phi_{j,t}\beta(1+(1-\tau_t^a)r_{t+1}/\phi_{j+1,t+1}) E_j v'_{j+1}(a', e', j+1; X_{t+1}, \Omega_{t+1}),$$
$$u_\ell(c_{j,t}, \bar{\ell} - \ell_{j,t}) \geq u_c(c_{j,t}, \bar{\ell} - \ell_{j,t})\,(1-\tau_t^{ss})\, w_t \eta_j e_j.$$

2.5 Definition of a Competitive Equilibrium

Our concern in this paper is the stationary state and the transition paths of the economy. Therefore, we need three definitions of equilibrium in the stationary state and in transition.

Definition 1 (Recursive Competitive Equilibrium) *Given the government's policy $\{\Omega_t\}$ and the population dynamics, the Recursive Competitive Equilibrium is a set of value functions $\{v_t\}$, policy functions $\{g_{c,t}, g_{\ell,t}, g_{a,t}\}$, aggregate capital $\{K_t\}$, aggregate labor $\{L_t\}$, factor prices $\{r_t, w_r\}$, payroll taxes $\{\tau_t\}$, and government funds $\{W_{g,t}\}$ that satisfy following conditions:*

(i) A Household's Optimality: Given the factor prices $\{r_t, w_r\}$ and payroll taxes $\{\tau_t\}$, the value function $\{v_t\}$ solves equation (6), and $\{g_{c,t}, g_{\ell,t}, g_{a,t}\}$ are the associated policy functions. The value and policy functions are measurable.

(ii) A Firm's Optimality: The factor prices are competitively determined as follows,

$$r_t = \theta A_t \left(K_t/L_t\right)^{\theta-1} - \delta,\;\; w_t = (1-\theta)\, A_t \left(K_t/L_t\right)^{\theta}.$$

(iii) Market Clearing: The market clearing conditions of equations (3) and (4) are satisfied.

(iv) the Government's Budget: For each case of $W_{g,t} > 0$ and $W_{g,t} = 0$, the governments' budget (5) clears.

(v) Transition Law: $\Phi_{t+1} = H(\Phi_t)$.

Definition 2 (Stationary Recursive Competitive Equilibrium) *The Stationary Recursive Competitive Equilibrium is a recursive competitive equilibrium with a stationarity of distribution $\Phi_{j,t+1} = \Phi_{j,t} (\forall t)$ for each age group j.*

[8] For details, see the Appendix.

Definition 3 (Balanced Growth Path) *A Balanced Growth Path is a recursive competitive equilibrium with* $Y_t = ((1+g)(1+n))^t Y_0$, $K_t = ((1+g)(1+n))^t K_0$, $W_{g,t} = ((1+g)(1+n))^t W_{g,0}$, $L_1 = (1+n)^t L_0$, $r_t = r_0$, $w_t = (1+g)^t w_0$, $c_{j,t} = (1+g)^t c_{j,0}$, $a_{j,t} = (1+g)^t a_{j,0}$, *and* $\ell_{j,t} = \ell_{j,0}$. *In other words, without a labor supply, the aggregate variables are normalized by* $A_t^{\frac{1}{1-\theta}} N_t$, *and individual variables are normalized by* $A_t^{\frac{1}{1-\theta}}$.

The final purposes of the paper to examine the macroeconomic and welfare implications of the competitive equilibrium on the transition path requires complex computational procedures. In this paper, we follow the method proposed by Conesa and Krueger (1999), who compute two stationary equilibria and the transition path by backward induction. On this transition path, all the markets clear and the above definition of competitive equilibrium is satisfied. To compute the transition, we need to calibrate initial and final stationary states. We set an initial stationary state in the year 2000 and a final state in the year 2200. In an actual numerical procedure, we compute a detrended path.

2.6 Policy Experiments

Under the transition path, there are mixed effects of allocation problems such as pure aging, increasing tax burden, and changing factor prices. In particular, the process of aging in retired households accelerates, and in one of population projections, the fraction of the retirees over the employees exceeds 40%. Thus, the aging of society might at least temporarily be considerably larger than in the final stationary equilibrium. Therefore, we need to consider allocations in the transition path from 2000 to 2200.

As a benchmark, we use the medium variant of the population projection by the National Institute of Population and Social Security Research (NIPSSR). Moreover, for policy experiments, we consider four social security reform plans. We compute the following scenarios:[9]

- As a *benchmark*, the replacement rate is targeted at 50%, i.e., $b(\tau_t, W_{g,t}) = 0.5$. We investigate the effects of pure aging, namely changes in the payroll tax burdens and factor price. We use the medium variant of the population projections and assume that the social security fund is zero.
- *Social Security Reform I*: Given zero social security fund, we consider a gradual cut of social security benefit to half; i.e., the final replacement rate is 25%.
- *Social Security Reform II*: Given a zero social security fund, we again consider a gradual cut of social security benefit to nearly-zero. For computational reason, we

[9] Conesa and Krueger (1999) consider three cases; (1) a sudden cut in social security benefit, (2)a gradual decrease in the replacement rate between 50 years, (3) cut in social security after 20 years.

set the replacement rate as 0.1% and not as zero. This scenario implies the full privatization or transition to the funded social security system.

- *Capital Income Tax*: We consider the other source of finance to sustain the social security system. First, we introduce s linear capital income tax to partly finance the social security benefits. The tax rate is set as 30%. In the tax rate, the remaining tax rate is approximately 5%, as mentioned later.

- *Consumption Tax*: Second, we introduce consumption tax to finance the remaining social security benefits. The consumption tax rate is set as 5%. We choose the tax rates is such that the remaining payroll tax rate is almost the same as approximately 5% of the tax rates.

Given these scenarios, we conduct a sensitivity analysis, considering the low and high variants of the population projections, and the effect of a positive government fund in the following section.

3 Calibration

3.1 Fundamental Parameters

We solve the model numerically because we have no closed-form solution. First, we calibrate the fundamental parameters in the model. As a target of the initial stationary state, we choose the Japanese economy in the year 2000.

The households enter into our economy at age 20, supply labor until age 65, and lives till age 100 at most(i.e., $J = 100$, $j_r = 65$). Although many workers compulsory retire at 60 in Japan, for the year 2000, our model contains a voluntary labor supply even over the age of 60. Even though we permit households to endogenously supply labor over age 60, a large fraction of the employees voluntary retires because of low productivity.

We assume that instantaneous utility function is of the Cobb=Douglas-type;[10]

$$u\left(c_{j,t}, \bar{\ell} - \ell_{j,t}\right) = \frac{\left[c_{j,t}^{\sigma}(\bar{\ell} - \ell_{j,t})^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}.$$

Following Conesa and Krueger (1999), the relative risk aversion parameter is set at $\gamma = 2$. This value is quite standard in the literature. Abe, Inakura, and Yamada (2007) estimate the fundamental parameters by structural estimation using Japanese

[10]Heathcote, Storesletten, and Violante (2005) investigate the importance of insurance on income risks when the utility function is separable and non-separable with regard to leisure. We use the non-separable utility function that is used in broad macroeconomics literature because we consider growth economy. Note that some empirical facts reveal that microeconomic behavior is consistent with the separable utility function, although it contradicts with a growing economy.

Panel Study of Consumers data compiled by the Institute of Household Economy, and find that the relative risk aversion parameter ranges from 2 to 7. A share parameter for consumption and leisure is set at $\sigma = 0.38$. This value is also close to the research by Abe et al. (2007). Moreover the average hours worked in the model matches the actual Japanese data using the parameter. Although Abe et al. (2007) estimate the discount factor and find that the estimate value ranges over $0.95 \sim 1.003$, the rage is somewhat broad to fix. Alternatively, as will be mentioned later, we use the capital-output ratio in the model as a target to determine the discount factor, $\beta = 0.984$, which is close to Hayashi and Prescott (2002) who calibrate the Japanese economy and set $\beta = 0.976$. We set $\bar{\ell} = 3.0$ so as to aggregate the labor supply in the model close to one.

Lastly, we choose the parameters for the production function. The capital share parameter, θ, is fixed at 0.312 to simultaneously match the capital-output ratio and interest rate in the stationary equilibrium in the year 2000. The depreciation rate is taken from Hayashi and Prescott (2002), and the value is specified at $\delta = 0.089$. Table 1 summarizes all the calibrated parameters.

3.2 Idiosyncratic Income Risk

It is difficult to estimate parameters for idiosyncratic income uncertainty that all households face because of the scarcity of micro data in Japan. Abe and Yamada (2006) is an exceptional work. Ohtake and Saito (1998) find that the logarithm of the variance of income in Japan increases across age groups. Moreover, they show that the shape of the age-variance profile is convex over age groups. To account for the convexity of the variance profile, Abe and Yamada (2006) specify the labor income process and estimate the parameters In this paper, we use the estimation shown in Appendix Table 2 in Abe and Yamada (2006). For incorporating the nonlinearity of the income variances, we use an age-dependent income variance shock.

As defined in Section 2.2.2, we assume that the idiosyncratic labor productivity process follows equation (1) and (2). Abe and Yamada (2006) report on the possibility of $\rho \geq 1$ because of the convexity of the variance profile. However, incorporating $\rho \geq 1$ makes the numerical computation far more difficult. Thus, we choose the persistence parameter to be close to one and standard deviation of the persistence shock increases across age groups(i.e., $\rho = 0.98$, $\sigma_{z_{20}} = 0.25$, $\sigma_{\varepsilon_{20}} = 0.0497$, and $\triangle\sigma_z = 0.001$). After the specification, we approximate the AR(1) process as 9-state Markov chain by using Tauchen's (1986) method.[11,12] The initial value of the persistent shock, z_{20}, is assumed to be one and the expected values for each age group are always one. The standard deviation of the transitory shock and the fixed effect are estimated to be $\sigma_\kappa = \sigma_\alpha = 0.135$. We approximate the transitory shock which has three states, namely $\exp \kappa = \{0.80, 1.07, 1.34\}$

[11] Flodén (2007) demonstrates that a modified version of Tauchen's method might be a better choice for approximating an AR(1) process with high persistence.

[12] For the details of the effect of the persistence parameter ρ on the variances of consumption and income profile, see Yamada (2007).

with probabilities $\{0.35, 0.57, 0.07\}$ respectively. In the benchmark model, we assume that there are no fixed effect because of the computational burden.[13] Moreover, we assume that the structure of the income risk does not change over the transition path. Figure 1 represents the cross sectional variances of the logarithms of consumption and earnings across age groups. Apparently, the simulated income profile traces the actual variance profile in Japan. On the contrary, the simulated consumption variance profile is much lower than the actual variance profile because we neglect the fixed effect.

3.3 Average Hourly Wage Profile

The efficiency unit of hourly wage for each age $\{\eta_j\}$ determines the average wage profile. We conduct the calculation following the method proposed by Braun et al. (2006) based the Report on the Special Survey of the Labor Force Survey by the Statistics Bureau, the Management and Coordination Agency, Government of Japan.[14] Table 2 presents the average hourly wage for each age group; we use a smoothed profile. The average hourly wage of the data is taken from the before-tax earnings. Although we do not include the income tax explicitly, which implies that the disposable income is large, our calibration procedure does no affect our results.

3.4 Demographic Structure

We choose demographic parameters to replicate the actual and projected population dynamics. The NIPSSR (2002) provides population projections from 2001 to 2050. We set the survival probability of $\{\phi_{j,t}\}_{t=2000}^{2050}$ from the value estimated by the NIPSSR. The data for the year 2000 are already realized, but those after 2001 are estimated values based on the NIPSSR's population projections. The fertility rate ψ_t is taken from each of the three projections: the high, medium and low variants. Because the population growth in our model is represented by the growth rate of 0-year old children, we use the ratio of the projected population of new born people between period t and $t+1$.

Since we need to compute two stationary states and the transition paths, we set the initial stationary state in the year 2000. Although the population changes from 2000 to 2050 following the projection by the NIPSSR, the population growth rate converges to zero after the transition. However, the convergence rate is slow and it takes approximately 100 years to reach a new stationary population distribution. Thus, we choose the final stationary state in 2200. Following Braun et al. (2006), we assume that the population growth rate converges to zero, $\psi_t = 0$, between 2050 and 2060.

One problem that arises here is how to choose an initial population distribution in the initial stationary state. Apparently, the actual population distribution in 2000 does

[13] We confirm that the fixed effect does not affect macroeconomic variables significantly. Conesa and Krueger (1999) also show that such fixed effect have a minor role for investigating social security reforms.

[14] For details on the calculation, see Hansen (1993) and the Appendix in Braun et al. (2006).

not seem to be stationary because of the existence of the baby-boomer generations as depicted in Figure 2. However, to compute the initial stationary state, a population distribution is required. Therefore, we assume that the households in our model believe that the actual population in 2000 is stationary.

The projection by the NIPSSR displays *three variants*: the high, medium and low population projections. We plot a fraction of the child population (under the age of 19), the working population ($20-65$), and retired households ($66-100$) in Figure 2. The total population in 2000 is normalized as one. In each projection, the fraction of retired households peaks in 2050, and thereafter, the rate converges to a new stationary state. In the low variant, the fraction of the retired households reaches over 40% and the working population sharply decreases with fewer birth.

3.5 Macroeconomic Variables as a Target

Finally, we choose the target macroeconomic variables to the calibrate Japanese economy. In Japan, the social security trust funds have two components; the fund from (1) Kokumin Nenkin Hoken ($9,800$ billion yen) and from (2) Kosei Nenkin ($136,900$ billion yen). Because Japan's GDP in 2000 was approximately $502,000$ billion yen, the targeted ratio for the social security fund in initial stationary state is set at $W_{g,t}/Y_t = 136.9/502.78$. In our model, the TFP growth rate is exogenously given and is perfectly forecastable. From Hayashi and Prescott (2002), Chen et al. (2006a) and Braun et al. (2006) the TFP growth is set at $\left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\theta}} = 1.01$, and A_0 is normalized as one.

4 Factor Prices and Social Welfare in a Transition Economy

4.1 Stationary State

To conduct our computational task, we need to know the stationary states of the initial and final periods. Before analyzing the transition path, we compare these stationary states. Table 3 and 4 summarize the equilibrium factor prices, the aggregate capital and labor, the payroll tax rate, the Gini coefficients and the macroeconomic statistics for each cases. These tables contain several facts on the aging society and social security reforms.

First, we confirm that the target macroeconomic variables of our model, the capital-output ratio and the equilibrium interest rate in the year 2000, are actually close to one in Japan, as shown in Hayashi and Prescott (2002). The K over Y is 2.42 and the equilibrium capital return is approximately 4% in our model. Based on the medium variant of the population projection, there is further capital deepening in 2200. The capital-output ratio increases by 3.11% and the interest rate decreases by 39 basis points.[15] In the benchmark case with the medium variant projection, only source for change of macro statistics is demographic structure, which also implies an increasing payroll tax rate. From Table 3 and 4, we confirm that the capital deepening is caused by the reduction of aggregate supply and not due to increased savings.

Second, the reduction of the replacement rate clearly promotes capital deepening further because of the strong incentive to save for retirement. For comparison, we compute a stationary equilibrium with the 25% and almost-0% replacement rate cases for the year 2000. If the social security system does not exist (or is a fully funded system) and all households must accumulate wealth for retirement, the capital-labor ratio is considerably higher than that in the benchmark case, and the equilibrium interest rate becomes 1.38%. Moreover, because capital deepening occurs in 2200, the equilibrium net interest rate will fall below 1% when the replacement rate is 0.1%.

Two tax reforms that provide another source of finance for social security benefit demonstrate different effects on households' decisions. Note that the consumption tax of 5% and the capital income tax of 30% implies that the remaining payroll tax is approximately 5%. However these taxes have opposite effects on labor supply decisions. When new financial resources are obtained from capital income tax, households increase their labor supply. On the contrary, they choose not to work more when a source of finance is obtained through consumption tax. Moreover, the aggregate labor does not move in the same direction as the number of hours worked, i.e. ch(L/N)$\neq$ch(hours). The aggregate labor does not increase as proportionately as the number of hours worked because the less efficient or more aged workers increase their labor supply. Therefore, even if consumption tax and capital income tax contribute the same to social security fund, they have opposite effects on macro statistics. In other words, tax reforms changes the labor profile and the substitution effect dominates the income effect.

Finally, we focus on wealth inequality because our model contains not only intergenerational but also intragenerational heterogeneity. The wealth Gini coefficient of total population and that of 30-65 is close to the one observed in Japan. Takayama and Arita (1994) show that the Gini coefficient of the total wealth inequality (Shomi Shisan) was estimated at 0.639 for the year 1989. Unfortunately, although the Gini coefficient of wealth inequality for the total population is close to the Japanese data, that in the same generation is smaller, especially in middle-aged and old households. Thus, as compared with the actual data, intragenerational wealth dispersion is equal if we adjust the income inequality based upon the estimation of Abe and Yamada (2006). A notable feature in the intragenerational inequality is that the introduction of the consumption tax induces high wealth inequality in young generations.

[15] The ch($\bullet$) in Table 3 and 4 represents percentage changes compared with the benchmark case in the year 2000.

4.2 Benchmark Case

We will begin by considering the transition path of equilibrium factor prices, payroll tax, output per capita, capital, labor and the Gini coefficients in the benchmark case, illustrated in Figure 3. The transition path does not seem to be monotonic due to the baby boomers and their children (baby boomer Jr.). As the economy has been aging for 50 years, there is a sharp decrease in the equilibrium interest rate owing to capital deepening. In contrast, the equilibrium wage increases up to 5% as compared to the case in the initial stationary state. According to the decrease in the interest rate, the Gini coefficient of the total population also reduces weakly over 50 years. The wealth inequality moves close to the interest rate, because a high interest rate implies a strong incentive to save. The high saving rate makes the wealthy households richer; however, unfortunately, the unlucky households who face a liquidity constraint cannot save further. Therefore, the high interest rate leads to wealth inequality. We discuss this point further in the following section.

To balance the government's budget in equation (5) for the social security system, the payroll tax rate must increase up to 18%. Although the output per capita decreases by 20% in the course of aging over 50 years and subsequently recovers weakly, there is more than one reason for output decreases. The rise of the capital-output ratio stems from the reduction in the per capita labor supply and a weak increase in asset holdings. A sharp rise in the payroll tax rate reduces the incentives to supply labor. On the contrary, the per capita asset increases because a fraction of near-retirement households increases as depicted in Figure 2. Since asset holdings reach a peak in around the age of 60 years, demographic changes affect the per capita capital. The labor supply of households tends to monotonically decrease because the labor supply is a decreasing function of asset holdings and due to the effect of aging. The wage increases also contribute to asset holdings for young households, which implies that such households prefer leisure in their old age. Therefore, in order to understand the pure effects of aging, we need to consider the age profiles with respect to labor supply and asset holdings. We present these profiles later.

4.3 Social Security Reform: Changes in the Replacement Rate

Sustaining current social security system causes a serious problem with respect to intergenerational equity. Specifically, the payroll tax creates a disincentive for labor supply and the output per capita decreases. To ease the increasing burden of the payroll tax, we consider two experimental cases: (1) the reduction of the replacement rate by half, and (2) a negligible replacement rate, i.e., 0.1%.[16] The latter case implicitly assumes the full privatization of the social security system or a transition to the funded system.

[16] Nishiyama and Smetters (2005b) carefully investigate the relationship between the privatization of social security system.and its efficiency gain.

First, we plot the transition paths of the macro variables with social security reform I in Figure 4. Capital deepening persists and intensifies for the first 50 years and the wage level correspondingly increases by 10% over the transition path. Accordingly, the wealth inequality decreases. The payroll tax rate does not exceed 12%, and the peak of the payroll tax rate does not correspond to aging. The curve depicting the output per capita is flatter than in the benchmark case since the per capita capital supply is larger by 15% and the aggregate labor does not decrease to great extent. Households supply their labor primarily in the case of a relatively small payroll tax and a high wage level. Moreover, asset accumulation for retirement improves the aggregate capital supply over a period of 40 years. In other words, future generations do not suffer as much from aging when measured in terms of per capita GDP due to both the reduction of the payroll tax and the upward trend of economy-wide wage level. We investigate the welfare implication of the social security reform in the following section.

The abolishment of the social security system results in substantial effects on the factor prices as depicted Figure 5. On the transition path, the negative real return on capital reduces further and the equilibrium wage sharply rises over 20%. As a result, the wealth inequality measured by the Gini coefficient for the total population also decrease sharply corresponding with the interest rate. The payroll tax rate peaks at 10%.

Middle-aged and old households hold greater assets because of the sharp rise of the equilibrium wage level. Owing to the lower payroll tax, the after tax earnings do not decrease even if the labor supply of young households is reduced. Not surprisingly, the per capita output increases for this transition path. Therefore, in this case, it can be said that the future generation receives benefits from these social security reforms. Note that unlike the young and future households at present, the middle-aged and old households do not necessarily benefit from this reform in 2000. This is similar to the problem of old households in the initial stationary sate in the usual overlapping generations model.

4.4 Capital Income Tax and Consumption Tax

For the last two scenarios, we consider the other financial schemes for social security reform. The government collects social security tax not only by means of the payroll tax but also by consumption or capital income tax. These financial sources of social security benefits are different from that of the payroll tax on labor income because all households including retired households are required to contribute to sustaining the social security system. As is already well known, the consumption tax does not distort the intertemporal consumption choice after the introduction of the tax system. On the contrary, the capital income tax affects disincentives for savings. From the above definition of social security reform, inducing capital deepening may be beneficial for young households. Thus, the capital tax may not improve welfare of the economy, especially of young households. However, recent research on capital tax reveals that good incentives for dissaving leads to increased labor supply and improves welfare.[17] Moreover, by using a model with idiosyncratic income risk and precautionary saving, Aiyagari (1995) shows that reducing the overaccumulation of capital could improves welfare. Therefore, we consider the consumption and capital income taxes.

First, we examine the case where in the government introduced linear capital income taxation in 2001, and thereafter, the linear capital tax rate is constant at 30%. All macro statistics are summarized in Figure 6. Since a part of the social security benefits is financed by the capital income tax, the maximum payroll tax rate does not exceed 16%, which is small as compared with the benchmark case. A notable feature of the capital income tax is that it has a relatively small effect on the factor prices path. While aging leads to capital deepening as depicted in Figure 3, the introduction of the capital taxation partially offset the capital deepening and corresponding factor prices become rather flat over 50 years. Both the aggregate capital and labor simultaneously decrease across the transition years and the Gini coefficient of wealth weakly increase over the years. Since the introduction of capital income tax is immediately terminated, the pre capita output sharply rises in 2001.

Next, we consider the case where the government introduces the consumption tax in 2001 as a financial source of the social security system, and subsequently, the case where the consumption tax rate is set at 5%. The transition path of macroeconomic statistics with consumption tax is close to the benchmark case as in Figure 7. The maximum payroll tax does not exceed 14%, which is considerably smaller than that of the benchmark case. Moreover, the payroll tax is lower than the case of capital income tax. Further, the output per capita is higher than the benchmark case. Therefore, both consumption and capital income tax improves the output per capita although both have different effects on factor prices.

5 Welfare Implications under the General and Partial Equilibrium Transition Paths

5.1 Evaluation of Welfare

For a comparison of the welfare implication of social security reform and aging, we need a criterion for evaluation the intergenerational and intracohort households. To evaluate the welfare of households, following Aiyagari and McGrattan (1998), we employ expected value of the initial period as follows:

$$Ev_t(a_{20}, s_{20}) = \sum \pi(s) v_t(0, s_{20}). \tag{9}$$

This criterion implies that we use a measure of the expected value of households who enter the economy in period t at age 20. In other words, it is a life time discounted

[17] For example, see Kocherlakota (2006).

value of each cohort before entering the economy. By assumption, the households have no wealth, but they realize some labor productivity of the fixed effect. This welfare measure also implies Rawls's "veil of ignorance". Note that the criterion does not actually measure households who have already entered the economy, i.e., the initial old households. To partially avoid the problem partially, we compute the cohort's welfare before the year 2000. Moreover, following Conesa and Krueger (1999), we compute the following welfare measure

$$EV(a_{20}, s_{20}) = \left(\frac{Ev_t^{\text{Reform}}(a_{20}, s_{20})}{Ev_t^{\text{Bench}}(a_{20}, s_{20})} \right)^{\frac{1}{\sigma(1-\gamma)}}, \tag{10}$$

which compare the consumption equivalent variation of cohorts between the benchmark and some social security reform.

Figure 8 depicts the expected value of each cohort using the welfare criteria of equation (9) and (10). The horizontal axis represents the year that a household enter the economy at age 20. In the benchmark case with the medium variant of the population projection and no social security reform, the cohort's welfare decreases for the aging period of 50 years and reaches the lowest point around 2050. A social security reform plan with replacement rate of 25% and 0.1% exacerbate the situation in new born households that have already entered the economy, and improves that of the younger and future generations. Note that households who enter the economy in year 2000 do not benefit from the reform because the burden is doubled with the transition to the funded system. On the contrary, the introduction of capital income tax weakly improves the welfare of the current young and future generation households. Although capital income tax improves the social welfare of the young and future cohorts, this does not imply that it results in a Pareto improvement since the standard overlapping generations model contains old households in the initial stationary state. A new financial scheme of social security financed by the consumption tax does not improves the welfare even of young and future households. There are two reasons for not improving the welfare. First, the consumption tax disperse the wealth inequality measured by Gini coefficients as described in Table 3. Such wealth inequality induces consumption inequality and declines the welfare of cohorts. Another reason for this is the substitution effect of the consumption profile. This is discussed in detail in later.

5.2 The Transition Path without Factor Price Adjustment

Before analyzing the details of the households behavior on the transition path, we need to consider the effect of factor price changes. Factor price dynamics constitute the key to understanding the dynamics of social security reform in our paper. To analyze the pure effect of social security reform, we need to eliminate the factor price dynamics. We investigate the transition dynamics with a fixed factor price path in a partial equilibrium path or small-open economy.[18] We fix the transition paths of factor prices to that of the benchmark case. The replacement rate and each taxes are the same as in each reform cases.

Figure 9 is a revision of Figure 8 with a factor prices path that is the same as that in the benchmark case. If there are no factor price changes, the welfare implication of the reforms change. If the replacement rate is reduced by the government, the old households do not suffer as much damage by this reform. Moreover, there is a weak improvement in the welfare of future generations measured by equation (10). The social security reform plan with capital income tax leads to a considerable improvement in the future generations' welfare relative to the case in a closed economy. The consumption equivalent variations increase by approximately 2.5%. On the other hand, the consumption tax does not improve their welfare even in this case though the reduction is small relative to the general equilibrium model. Generally, the factor prices are affected by several factors, for example, monetary policy, foreign capital flows, immigration, and changes in capital or labor intensive technology. Therefore, the partial equilibrium path may be the possible case.

5.3 Each Profiles under the Transition Path

Figure $10 - 12$ plots a cross sectional profile of earnings, assets, and consumption. In the benchmark case, although the earnings profiles do not change considerably over the transition, middle-aged households consume more whereas retirees consume less. Thus, the consumption profile acquires a more defined inverted u-shaped. In other words, households substitute consumption at a young age for that at an old age. Further, the asset profile monotonically decreases over age.

Two social security reforms have the same qualitative features although their magnitude differs. When the replacement rate is reduced to 25% or 0.1%, two opposite effects exist; income and substitution effects of labor supply. Under the transition path of the reform, middle-aged and old households would want to hold more assets around their retirement. Because substitution effect dominates the income effect, young households consume more; in fact they are almost hand-to-mouth consumers. Thus, they have almost zero assets. In their middle age, they work more and accumulate wealth rapidly. Therefore, under the social security reform, the aggregate labor increases because the middle-aged and old households increase their labor supply considerably. Because households prefer consuming in the earlier stage of their lives, their welfare increases with positive discounting.

These figures also reveals why the consumption tax does not improve the households' welfare. Both consumption tax and capital income tax weakly expand the labor sup-

[18] We do not explicitly consider an open economy although international capital flow have strong effects on capital return. Krueger and Ludwig (2006) and Attanasio, Kitao, and Violante (2007) investigate the transition path of factor prices using a multi-country model.

ply profile. However, consumption and asset profiles of those case differ each other. In the transition path with capital income tax, the asset profiles monotonically decreases. In comparison, in the transition path with consumption tax, a household prefers to accumulate greater assets and consumes less. However, with lower interest rate, the consumption profile with consumption tax is lower than that with capital income tax. Moreover, the introduction of consumption tax leads the households consume more in the early stage of lifecycle. As mentioned before, the consumption inequality of young generation is larger in the case of consumption tax than that in the benchmark. Therefore, the expected value is lower in the case of consumption tax measured by equation (9). Nishiyama and Smetters (2003) also show that if there is no idiosyncratic risks, introduction of the consumption tax as a new source of finance for the government expenditure improves welfare of the economy, on the contrary, if the shock is uninsurable, such reform reduces efficiency.

6 Robustness Check

6.1 Various Estimations on the Population Projections

Our results are based on the medium variant of the population projection obtained from the NIPSSR. Although the population projection follow a precise demographic viewpoint, the actual population dynamics in recent years in Japan are rather close to the low variant scenario. Thus, we perform a robustness check by reporting the transition path with the low and high variant.

Figures 13 and 14 reveal the transition path with the low and high variants respectively. Not surprisingly, if the speed of aging in these variants is quicker than in the medium variant, there is greater capital deepening. If the low variant is realized, the payroll tax rate exceeds 20%, which contradicts with the current system in Japan. Correspondingly, the output per capita decreases by over 20%. The governor in Japan claims that the payroll tax will be fixed a maximum of 18.30%. Thus, some social security reform is required.

6.2 Positive Government Funds

The social security system in Japan in year 2000 has positive government funds as stated in Section 2. In order to sustain the actual social security system in Japan, we cannot escape from an increasing payroll tax or the reduction of the replacement rate. To ensure the fairness of intergenerational allocation, a possible scenario is that where a weak reduction in the government's fund relieves such intergenerational inequality. We consider the case where the government cuts down the social security funds for the 50th year of the aging period.

When the government gradually cut down the fund, the payroll tax over the transition becomes smaller than that in the benchmark. Households allocate most of the increase in the disposable income to savings, which implies that the capital-output ratio increases as compared with the benchmark as depicted in Figure 15. Therefore, even though there is some government fund, it is not sufficient for the improvement of intergenerational inequality.

7 Concluding Remarks

We have investigated the macroeconomic and welfare implications of aging in Japan. When the low variant of population projection realized, some reform must be inescapable. Therefore, we consider four social security reform plans: the reduction of the replacement rate to 25%, full privatization, capital income tax, and consumption tax. For our criteria of social welfare, we find that capital income tax weakly improves the young and future cohorts' welfare. Moreover, consumption tax should not necesarily improves the welfare, though it increases the per capita output, because of intragenerational heterogeneity. On the other hand, it is not surprising that social security reform with changes in the replacement rate leads to intergenerational conflicts between the old and future generations. As compared with the introduction of capital income tax, partial privatization will improves the welfare of future cohorts.

An important issue to be considered in implementing these reforms is the change in factor prices and heterogeneity over the transition path. To a sizeable extent, a social security reform should effect a change in the behavior of households with respect to savings that will, in turn, change the capital-output ratio. Such effects may enlarge or offset the effect of social security reform. In our experiments, if the factor price adjustment is sufficiently effective, the introduction of capital income tax is a more favorable option. To understand the effect of aging and social security reforms, we focus on each profiles under transition. The key to understand this effect is the shape of consumption profiles curves. If the changes in these factors are disregarded, it will lead to a misunderstanding of the aging effect and social security reform.

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A Details of the Model and Numerical Procedures

A.1 Normalization

Because we consider growing economy with population dynamics, we need to detrend aggregate and individual variables. The Bellman equation after normalization of population and TFP growth are as follows:

$$v_t(\tilde{a}_j, s_j; X_t, \Omega_t) = \max_{\tilde{c},\tilde{a}'} \left\{ u(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t}) + \phi \tilde{\beta}_t E v_{t+1}(\tilde{a}', e'; X_{t+1}, \Omega_{t+1}) \right\}$$

subject to

$$(1+\tau_t^c)\tilde{c}_{j,t} + (1+g_t)\tilde{a}_{j+1,t+1} = (1+(1-\tau_t^a)r_t/\phi_{j,t})\tilde{a}_{j,t} + (1-\tau_t^{ss})\, w_t \eta_j e_j \ell_{j,t},$$
$$(1+\tau_t^c)\tilde{c}_{j,t} + (1+g_t)\tilde{a}_{j+1,t+1} = (1+(1-\tau_t^a)r_t/\phi_{j,t})\tilde{a}_{j,t} + w_t b(\tau_t, W_{g,t}),$$

where $\tilde{\beta}_t = \beta(1+g_t)^{\sigma(1-\gamma)}$, $c_{j,t}/(A_t^{\frac{1}{1-\theta}} N_t) = \tilde{c}_{j,t}$ and $a_{j,t}/(A_t^{\frac{1}{1-\theta}} N_t) = \tilde{a}_{j,t}$. Under the balanced growth path, aggregate variables $(Y_t, K_t, W_{g,t})$ grow at the rate of $(1+g_t)(1+n_t)$, and the growth rate of aggregate labor L_t is $(1+n_t)$.

A.2 First-Order Conditions

From the first-order conditions of the Bellman equation, we obtain

$$u_c'(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t}) - \lambda(1+\tau_t^c) = 0,$$
$$-u_c'(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t}) \frac{(1+g_t)}{(1+\tau_t^c)} + \phi_{j,t}\tilde{\beta}_t E_j \frac{\partial v_{t+1}(a', s')}{\partial a'} \leq 0,$$
$$\frac{\partial v_t(a_j, s_j)}{\partial a} = \frac{(1+(1-\tau_t^a)r_t/\phi_{j,t})}{(1+\tau_t^c)} u_c'(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t})$$
$$-u_\ell'(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t}) + \lambda(1-\tau_t^{ss}) w_t \eta_j e_j = 0,$$

where λ is a Lagrange multiplier on a budget constraint.

From the Envelope Theorem, intertemporal and intratemporal first-order conditions are as follows;

$$u_c'(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t}) \frac{(1+g_t)}{(1+\tau_t^c)} = \phi_{j,t}\tilde{\beta}_t E_j \frac{\left(1+(1-\tau_{t+1}^a)r_{t+1}/\phi_{j+1,t+1}\right)}{(1+\tau_{t+1}^c)} u_c'(\tilde{c}_{j+1,t+1}, \bar{\ell} - \ell_{j+1,t+1}),$$

$$\Rightarrow \frac{[\tilde{c}_{j,t}^\sigma (\bar{\ell} - \ell_{j,t})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j,t}} \frac{(1+g_t)}{(1+\tau_t^c)}$$
$$= \phi_{j,t}\tilde{\beta}_t \frac{(1+(1-\tau_{t+1}^a)r_{t+1}/\phi_{j+1,t+1})}{(1+\tau_{t+1}^c)} E_j \left\{ \frac{[\tilde{c}_{j+1,t+1}^\sigma (\bar{\ell} - \ell_{j+1,t+1})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j+1,t+1}} \right\},$$

$$\frac{u'_\ell(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t})}{(1-\tau_t^{ss})w_t\eta_j e_j} = \frac{u'_c(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t})}{(1+\tau_t^c)},$$

$$\Rightarrow (1-\sigma)\frac{[\tilde{c}_{j,t}^\sigma(\bar{\ell} - \ell_{j,t})^{1-\sigma}]^{1-\gamma}}{\bar{\ell} - \ell_{j,t}} = \sigma\frac{[\tilde{c}_{j,t}^\sigma(\bar{\ell} - \ell_{j,t})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j,t}}\frac{(1-\tau_t^{ss})w_t\eta_j e_j}{(1+\tau_t^c)}.$$

Then, from the intratemporal first-order conditions, labor supply function is

$$\ell_{j,t} = \max\left[\bar{\ell} - \frac{(1+\tau_t^c)}{(1-\tau_t^{ss})w_t\eta_j e_j}\left(\frac{1-\sigma}{\sigma}\right)\tilde{c}_{j,t}, 0\right],$$

where $\ell_{j,t} \in \left[0, \bar{\ell}\right]$.

A.3 Endogenous Gridpoint Method

Although there are many numerical methods for computing the policy function, we apply the Endogenous Gridpoint Method by Carroll (2006) because it is a relatively safe and faster method.[19]

Define the right hand side of the Bellman equation as

$$\Gamma_t(\tilde{a}_j, s_j) = \phi_{j,t}\tilde{\beta}_t E_{j+1} v_{t+1}(\tilde{a}', s'),$$

$$(1+\tau_t^c)\Gamma'_t(\tilde{a}_j, s_j) = \phi_{j,t}\tilde{\beta}_t\left(1 + (1-\tau_{t+1}^a)r_{t+1}/\phi_{j+1,t+1}\right)E_{j+1}u'_c(\tilde{c}_{j+1,t+1}, \bar{\ell} - \ell_{j+1,t+1}),$$

and take discretized grids on $\tilde{a}' \in [\underline{a}, \overline{a}]$. We set the number of grids to be 100. From above, the intertemporal first-order condition is rewritten as follows.

$$\frac{u'_c(\tilde{c}_{j,t}, \bar{\ell} - \ell_{j,t})}{(1+\tau_t^c)} = \frac{\Gamma'_t(\tilde{a}_j, s_j)}{(1+g_t)(1+\tau_{t+1}^c)}. \tag{11}$$

Thus, we can compute Γ'_t for each discretized state $(\tilde{a}_j, s_j)$. After taking inverse of the utility function, we obtain consumption $\tilde{c}_j$ for each state. From the Envelope Theorem and marginal utility function, the first-order condition is

$$u'_c(\tilde{c}_{j+1,t+1}, \bar{\ell} - \ell_{j+1,t+1}) = \sigma\frac{\left[\tilde{c}_{j+1}^\sigma\left(\bar{\ell} - \ell_{j+1}\right)^{1-\sigma}\right]^{1-\gamma}}{\tilde{c}_{j+1}}.$$

Suppose that next period's consumption and labor supply function is already known as

$$c_{j+1,t+1} = g_{c,j+1}(\tilde{a}_j, s_j),$$
$$\ell_{j+1,t+1} = g_{\ell,j+1}(\tilde{a}_j, s_j), \text{ if } j \le j_r,$$
$$\ell_{j+1,t+1} = 0, \text{ if } j > j_r.$$

then, by backward induction, we can compute the $\Gamma'(\tilde{a}_j, s_j)$ for each grids $\{a_{j,t}^{i\prime}\}_{i=1}^{n_w}$ for each age.

[19] For details on the endogeneous gridpoint method with endogenous labor supply, see appendix in Krueger and Ludwig (2006).

A.4 Inverse of the Utility Function

From the first order condition (11), by taking inverse of the utility function $u'_c(\tilde{c}_{j,t}, \bar{\ell}-\ell_{j,t})$ with respect to $\tilde{c}_j$, we obtain $\tilde{c}_j$ for each choice variable $\tilde{a}_j$. Using the Euler equation for leisure and removing $\ell_{j,t}$, we have[20]

$$u'_c(\tilde{c}_{j,t}, \bar{\ell}-\ell_{j,t}) = \tilde{c}_j^{-\gamma}\sigma\left(\left(\frac{1-\sigma}{\sigma}\right)\frac{1-\tau_t^c}{(1-\tau_t^{ss})w_t\eta_j e_j}\right)^{(1-\sigma)(1-\gamma)}.$$

This equation is apparently invertible. Thus, we have

$$\tilde{c}_j^i = u^{-1}\cdot\left(\frac{\Gamma'_t(\tilde{a}_j, s_j)(1+\tau_t^c)}{(1+g_t)(1+\tau_{t+1}^c)}\right).$$

After that we can directly induce ℓ_j^i. From the set of $\{\tilde{c}_j^i, \ell_j^i, \tilde{a}_j^i\}$, we have new cash on hand $\tilde{x}_j^i = (1+g_t)\tilde{a}_j^i + \check{c}_j^i$, where $\check{c}_j^i \equiv \tilde{c}_j^i + (1-\tau_t^{ss})w_t\eta_j e_j(\bar{\ell}-\ell_j^i)$.

A.5 Transition Function

From the policy function and transition probability of labor productivity $\pi(z'|z)$, transition function $Q_t(\cdot,\cdot)$ of states(a,s) and distribution function over the states can be computed. Define the probability space as $((A\times Z\times\mathcal{J}), \mathcal{B}((A\times Z\times\mathcal{J})), \Phi_j)$ where $\mathcal{B}((A\times Z\times\mathcal{J}))$ is a Borel σ-field and $\Phi_t(S)$ is a probability measure over $S\in\mathcal{B}((A\times Z\times\mathcal{J}))$. The probability measure is defined over individual state and also represents fraction of households with state $S\in\mathcal{B}((A\times Z\times\mathcal{J}))$. Because we assume that household of age $j=0$ have zero asset, Φ_1 is equal to one on $a_{1,t}=0$. The transition function $Q_j : (A\times Z\times\mathcal{J})\times\mathcal{B}((A\times Z\times\mathcal{J}))) \to [0,1]$ is defined as

$$Q_j((A\times Z\times\mathcal{J}), S) = \sum_{e'\in B}\begin{cases}\pi(z'|z) & \text{if } g_{a,t}(a_j, s_j)\in S\\ 0 & \text{else}\end{cases}, \text{ for all } j=1,\dots,J.$$

From the initial distribution $\Phi_{1,t}$, distribution function $\{\Phi_{j,t}\}_{j=1}^J$ for each j transit by the following equation.

$$\Phi_{j+1,t+1}(S) = \int Q_j((A\times Z\times\mathcal{J}), S)\, d\Phi_{j,t}, \ \ (\forall B\in\mathcal{B}((A\times Z\times\mathcal{J}))), \ j=1,\dots,J,$$
$$\Phi_{t+1} = H(\Phi_t)$$

Population dynamics is adjusted by μ_t, and the growth of TFP is already included. Thus, this distribution is purely a wealth distribution for each generation.

[20] Without labor supply, we can compute the inverse of the utility function from the following equation:

$$\sigma\frac{[\tilde{c}_j^\sigma(\bar{\ell}-\tilde{\ell}_j)^{1-\sigma}]^{1-\gamma}}{\tilde{c}_j} = \sigma\tilde{c}_j^{\sigma(1-\gamma)-1}\bar{\ell}^{(1-\sigma)(1-\gamma)}.$$

A.6 Computation of Steady State

Computation of the stationary state is the same as in Aiyagari (1994) model. Though there are three markets, the factor prices are determined from capital-labor ration K/L. Moreover, by the Walras law, we concentrate on K/L and governemnt budget celaring of τ^{ss}.[21]

1. Given initial guess of (K^0, L^0), compute Y^0. We also need initial guess of C^0 for consumption tax. From calibration of the social security fund, compute W_g^0.
2. Given $(r^0, w^0, K^0, L^0, C^0, W_g^0)$ and exogenous (τ^c, τ^a), compute the payroll tax rate τ_0^{ss} from the government budget condition.
3. Given $\left(r^0, w^0, \tau^{ss,0}; \tau^c, \tau^a\right)$, compute the policy function using the EGM and get distribution function Φ^0 for each age.
4. Integrating the distribution function Φ^0, get the aggregate capital and labor $\left(K^1, L^1\right)$.
5. If new $\left(K^1, L^1\right)$ and old (K^0, L^0) are sufficiently close to each other stop it, and we have equilibrium prices for given $\tau^{ss,0}$.
6. From new equilibrium condition $(r^1, w^1, K^1, L^1, C^1, W_g^1)$, re-compute a new payroll tax $\tau^{ss,1}$. Repeat step $3-5$. If iteration error of τ^{ss} is sufficiently small, stop it. If criterion of step 5 and 6 are satisfied, we have an equilibrium.

Note that all computation above are already detrended by $A_t^{\frac{1}{1-\alpha}} N_t$.

A.7 Transition Dynamics

Aftre computation of the steady state, we compute transtion path of each equilibrium. Basic idea here is the same as Conesa and Krueger (1999) and Nishiyama and Smetters (2004,2005a).

1. Guess future period T. We assume $T = 201$.
2. Set exogenous pair of tax rates (τ_t^c, τ_t^a).
3. Guess an equilibrium sequence of $\{r_t, \tau_t^{ss}, L_t, W_{g,t}, \xi_t; b(\tau_{ss,t}, W_{g,t}), A_t\}_{t=1}^T$ among stationary states.[22] Note that we implicitly assume that benefit of social security and the sequence of TFP $\{b(\tau_t^{ss}, W_{g,t}), A_t\}_{t=1}^T$ is perfect foresight and exogenously given. Thus, we have $\{r_t, w_t, \tau_t^{ss}, b(\tau_{ss,t}, W_{g,t}), \xi_t\}_{t=1}^T$ which is required to compute policy function.

[21] We take 100 grids on asset a for computing policy function, and to compute the distriution we take 5000 grids.

[22] For simplicity, we start linear case.

4. Because already we know policy function of final stationary state in period T, given sequences of step 2, compute the policy function from T to $t = 1$ by backward induction.

5. Using the obtained policy function, compute the sequence $\{K_t, w_t, W_{g,t}, b(\tau_t^{ss}, W_{g,t})\}_{t=1}^{T}$ from $t = 1$ forwardly.

6. Check whether each market clearing conditions and government budget balances are satisfied. If those are not in equilibrium, up-date the price sequences and repeas step $3 - 5$.[23]

7. If all markets clear in all periods, stop computation.

[23]There are many efficient methods for update the price sequence. For example, Krueger and Ludwig (2006) and Ludwig (2006) uses a modified version of Gauss-Zeidel method for computing the transition path.

B Figures and Tables

β	γ	σ	θ	δ	J	j_r
0.985	2.0	0.38	0.312	0.089	100	65

Table 1: Fundamental Parameters

age	hourly wage	age	hourly wage
$20-24$	$1,311$	$45-49$	$3,097$
$25-29$	$1,728$	$50-54$	$3,073$
$30-34$	$2,120$	$55-59$	$2,918$
$35-39$	$2,575$	$60-64$	$1,948$
$40-44$	$2,919$	$65-$	$1,655$

Table 2: Average Efficiency for Each Age; yen

	Medium Variant		Rep. Rate		Tax Reform	
	no fund	with fund	25%	0.1%	$\tau^c = 5\%$	$\tau^a = 30\%$
K/Y	2.42	2.48	2.63	3.03	2.45	2.24
K/L	3.61	3.75	4.07	5.02	3.69	3.22
ch(K/Y): %	–	2.69	8.72	25.53	1.54	−7.49
r (%)	4.01	3.67	2.97	1.38	3.81	5.05
w	1.03	1.04	1.07	1.14	1.03	0.99
τ^{ss} (%)	10.17	9.04	5.09	0.02	4.99	5.25
K/N	3.50	3.65	4.10	5.36	3.58	3.14
L/N	0.97	0.97	1.01	1.07	0.97	0.97
ch(L/N): %	–	0.25	3.78	9.97	0.09	0.52
ch(hours): %	–	0.41	4.35	11.52	−0.04	0.74
Y/N	1.45	1.47	1.56	1.76	1.46	1.40
Gini (20-100)	0.596	0.609	0.590	0.583	0.605	0.611
Gini (30-65)	0.531	0.545	0.549	0.565	0.543	0.548
Gini (20s)	0.586	0.587	0.591	0.605	0.643	0.588
Gini (30s)	0.589	0.580	0.586	0.589	0.634	0.580
Gini (40s)	0.393	0.417	0.420	0.443	0.409	0.424
Gini (50s)	0.263	0.274	0.254	0.232	0.267	0.276
Gini (60s)	0.303	0.316	0.238	0.171	0.302	0.314

Table 3: Stationary State Comparison in 2000

	Medium	Rep. Rate		Tax Reform	
	Variant	25%	0.1%	$\tau^c = 5\%$	$\tau^a = 30\%$
K/Y	2.49	2.76	3.28	2.54	2.32
K/L	3.77	4.39	5.62	3.87	3.40
ch(K/Y): %	3.11	13.96	35.59	4.90	−3.91
r (%)	3.62	2.42	0.62	3.40	4.53
w	1.04	1.09	1.18	1.05	1.01
τ^{ss} (%)	14.04	7.02	0.03	8.77	9.45
K/N	3.32	3.99	5.44	3.40	3.00
L/N	0.88	0.91	0.97	0.88	0.88
ch(L/N): %	−9.31	−5.75	−0.14	−9.28	−9.00
ch(hours): %	1.31	5.98	13.69	1.25	1.94
Y/N	1.33	1.45	1.66	1.34	1.29
Gini (20-100)	0.608	0.589	0.589	0.618	0.624
Gini (30-65)	0.555	0.592	0.592	0.567	0.571

Table 4: Stationary State Comparison in 2200

Figure 1: Variance Profiles

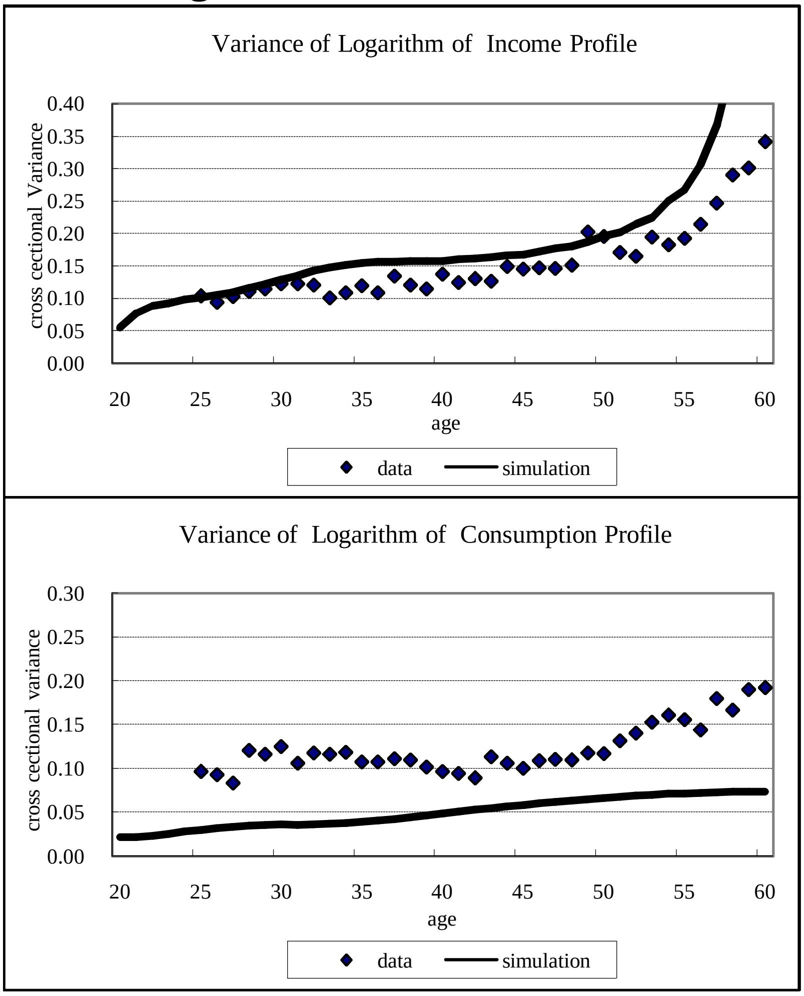


Figure 2: Population Dynamics in Japan

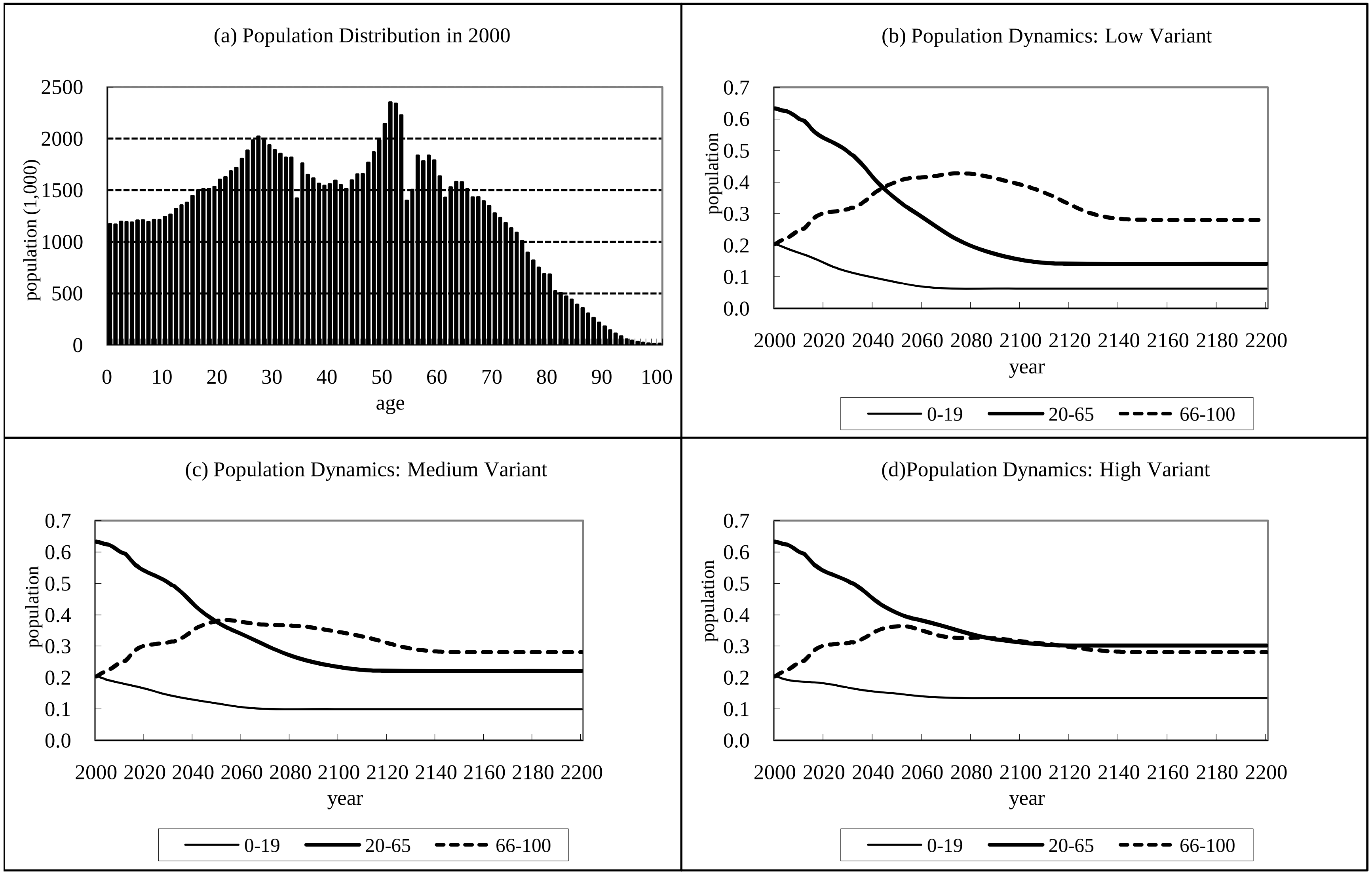


Figure 3: Benchmark Case (Medium Variant)

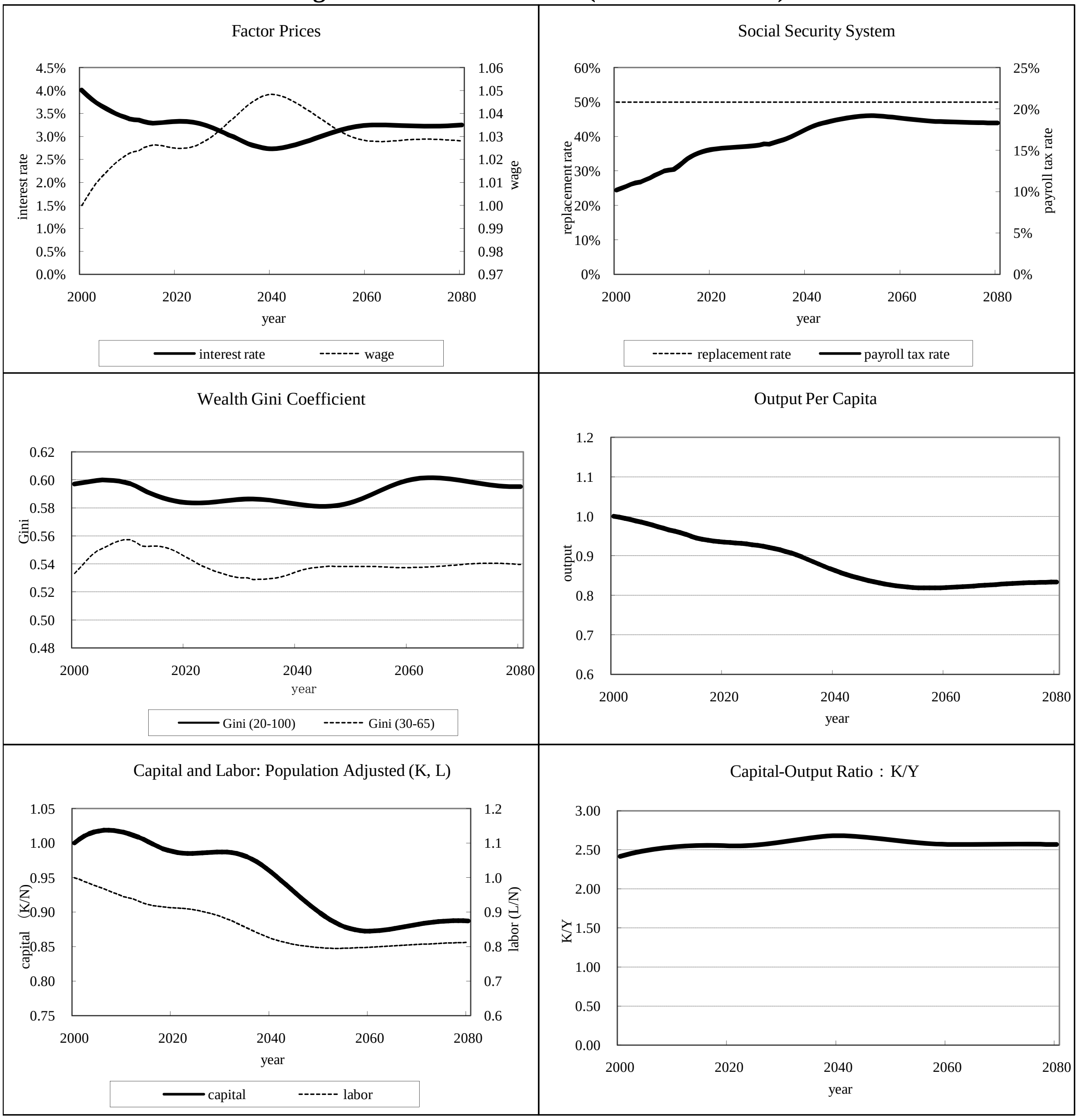


Figure 4: Social Security Reform I (25%)

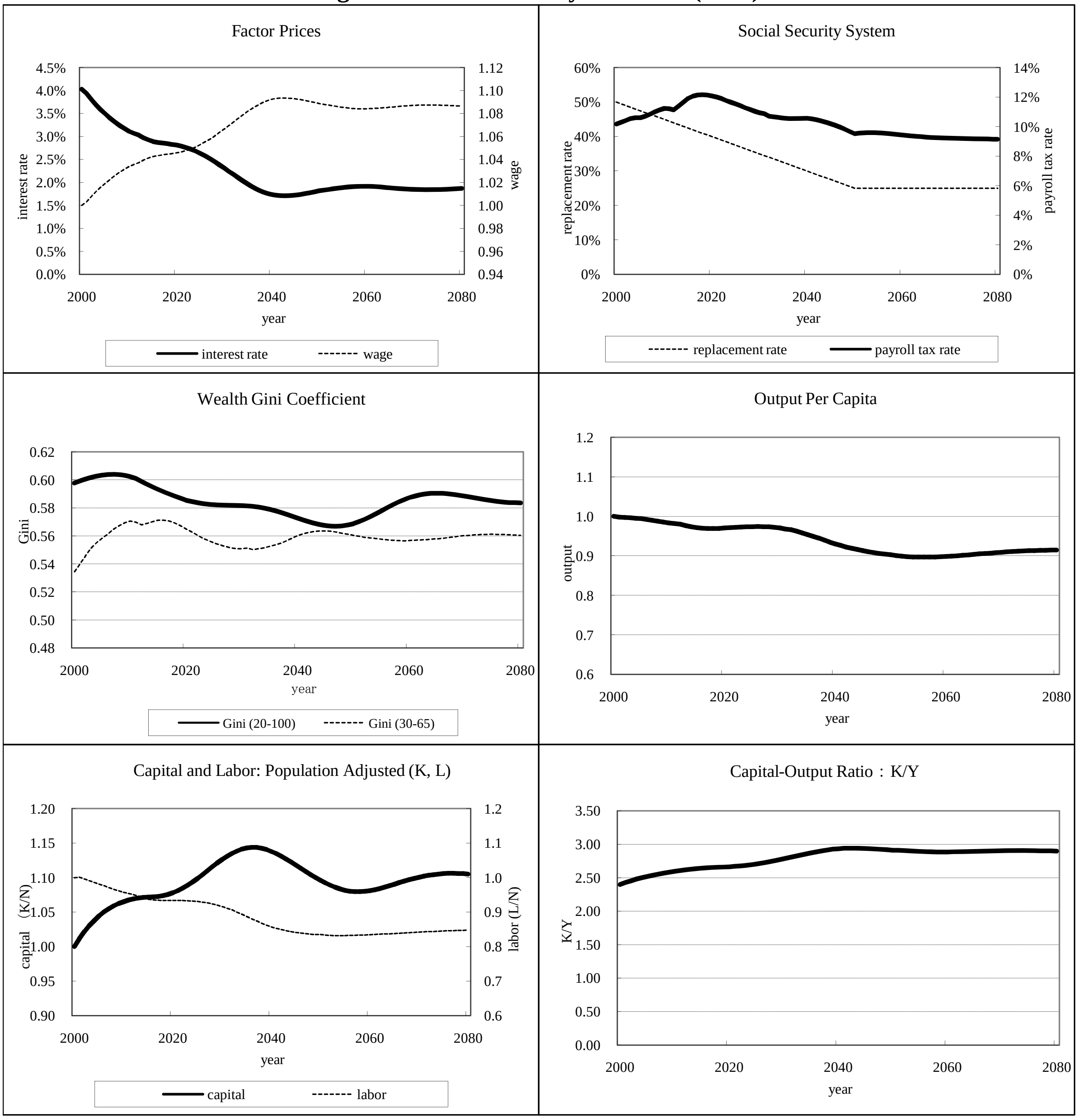


Figure 5: Social Security Reform II (0.1%)

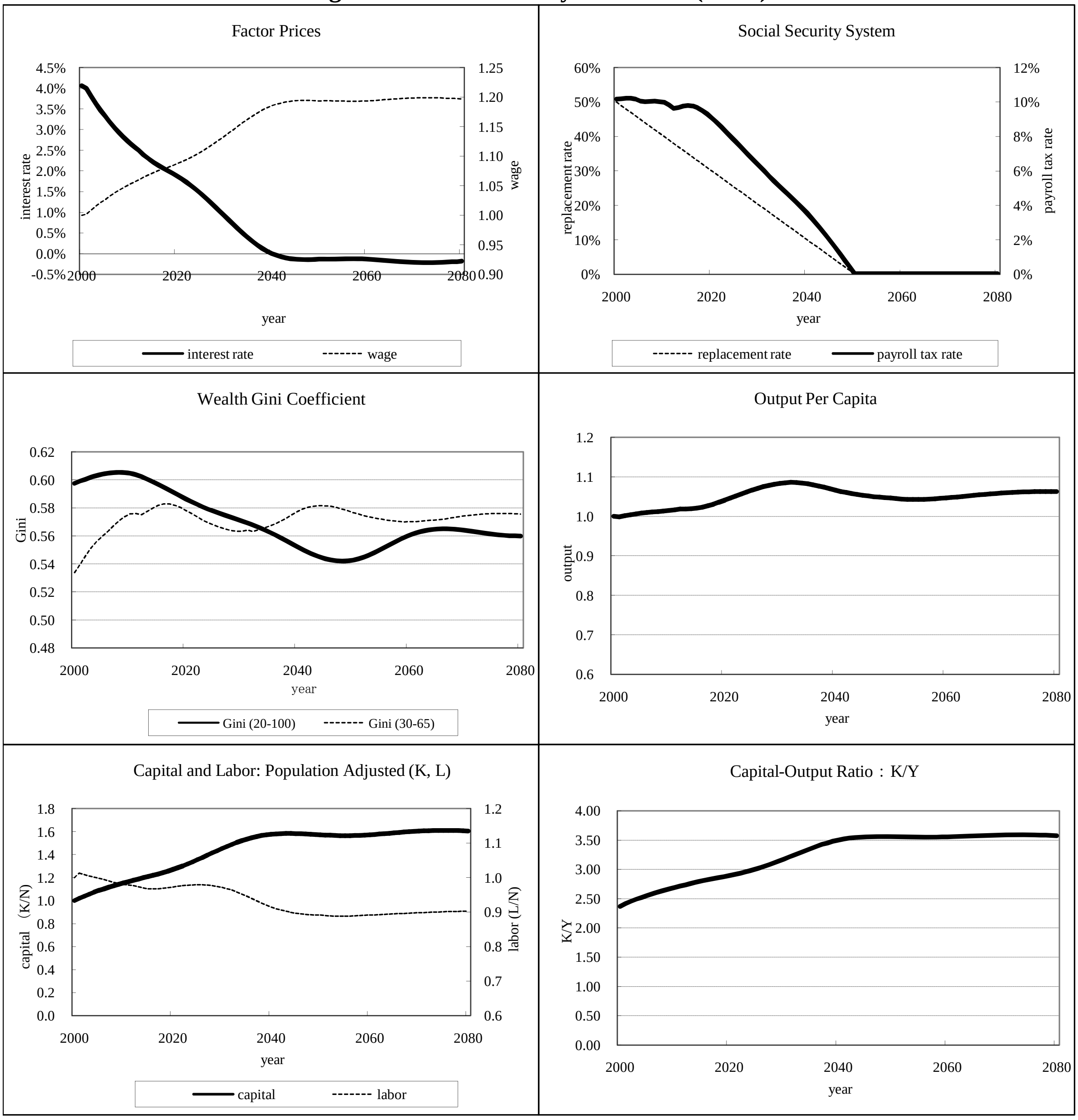


Figure 6: Capital Income Tax

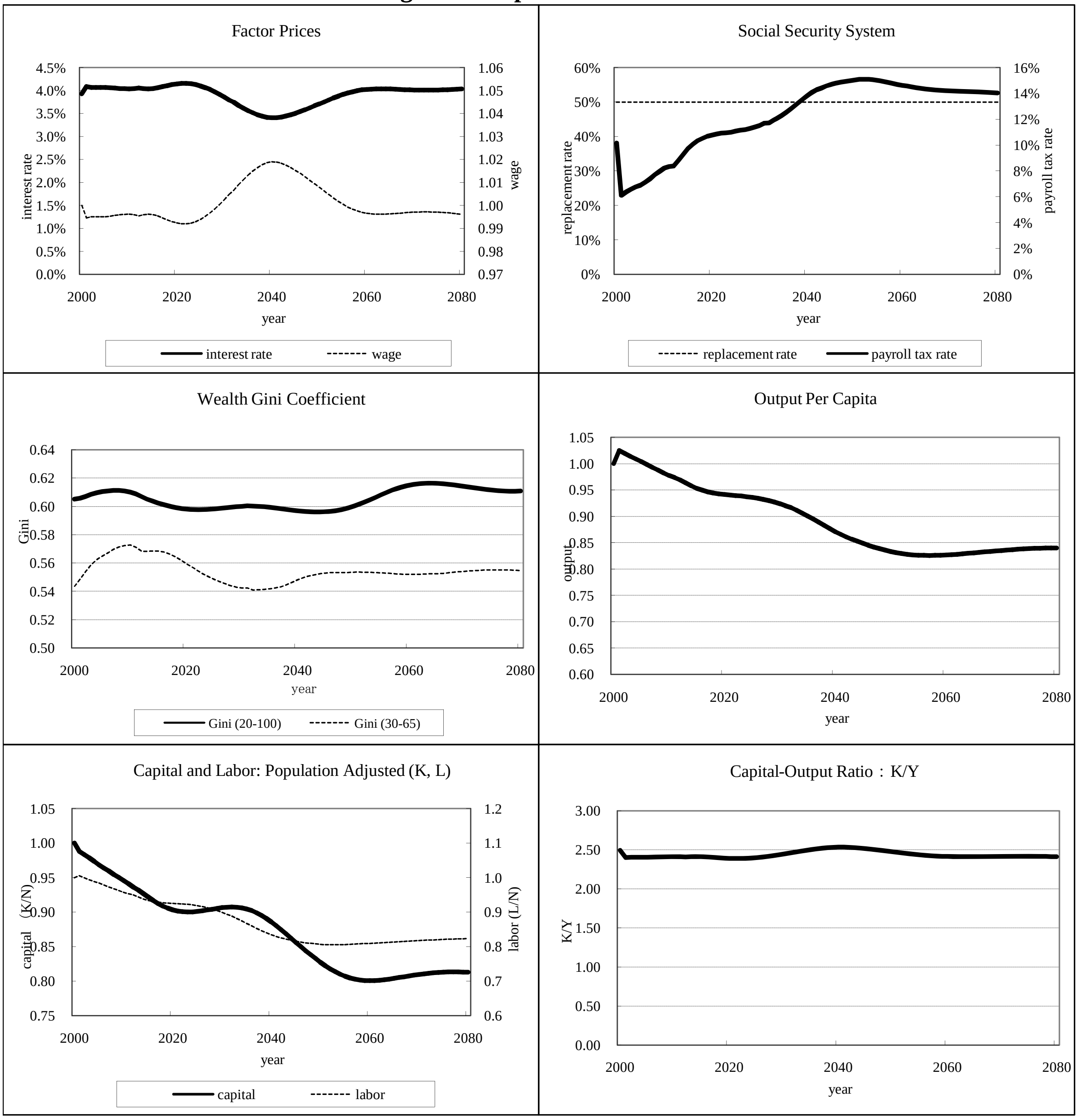


Figure 7: Consumption Tax

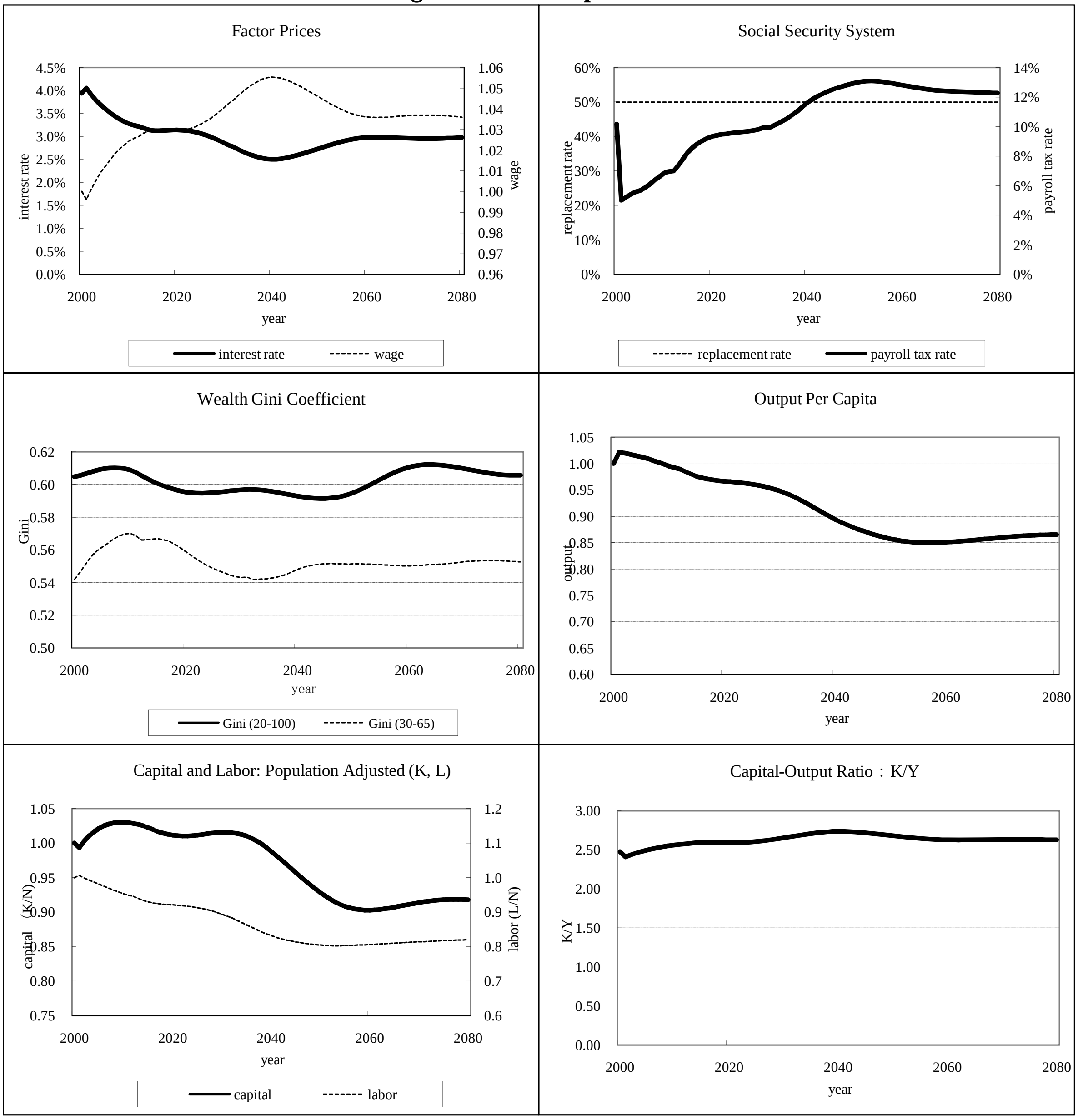
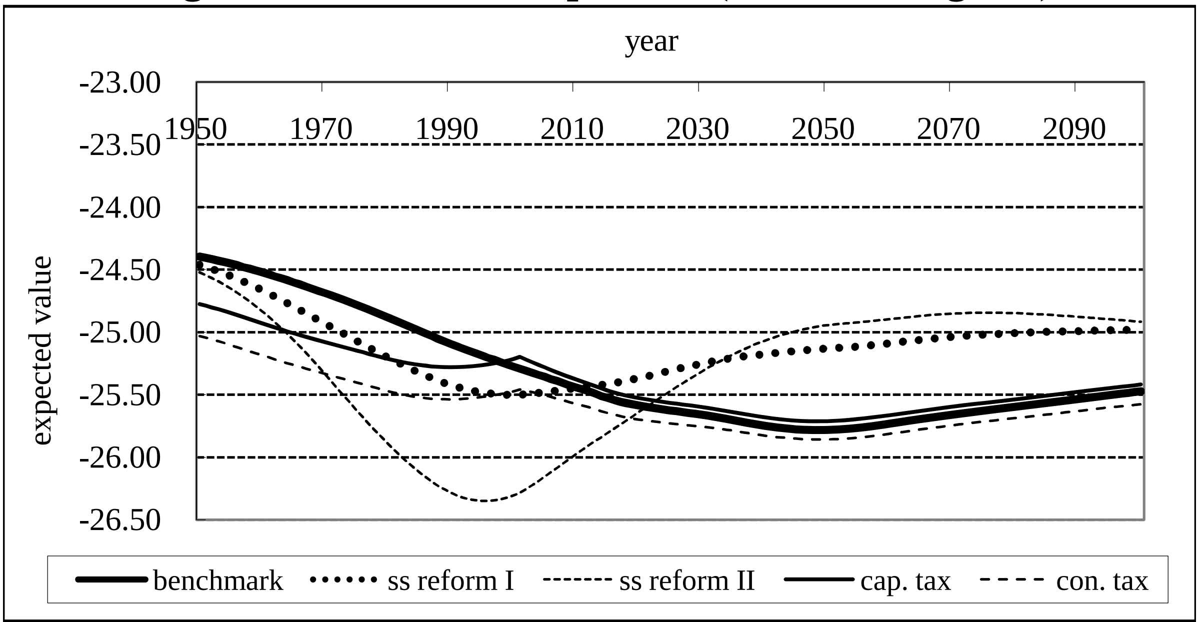


Figure 8: Welfare Comparison (Cohort at Age 20)



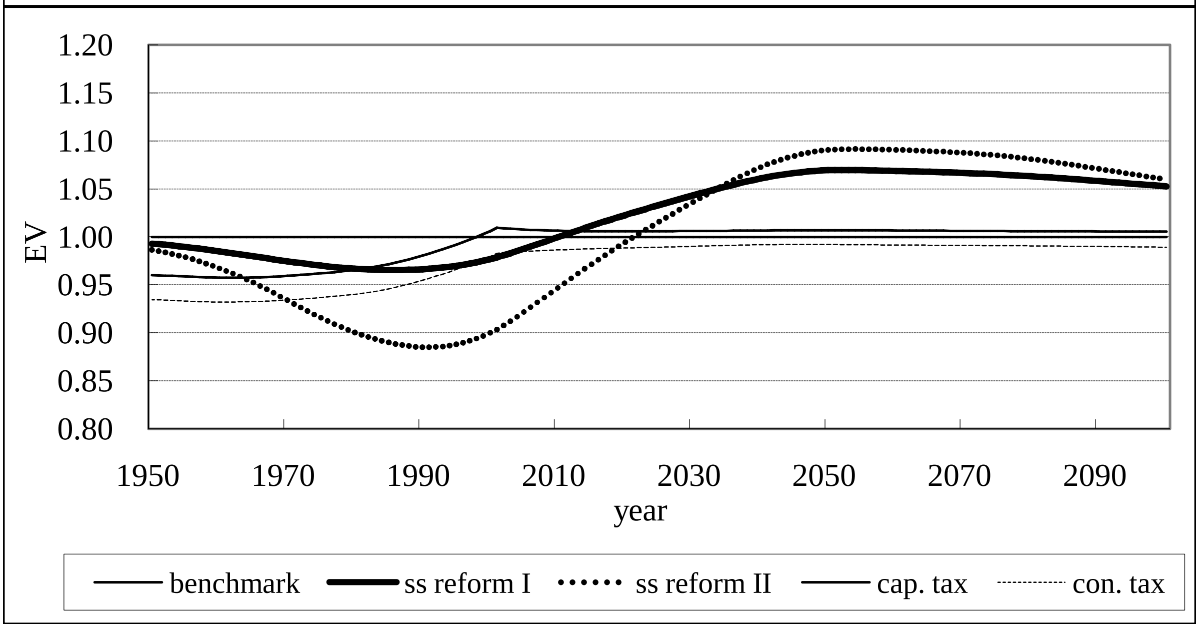


Figure 9: Welfare Comparison (Partial Equilibrium)

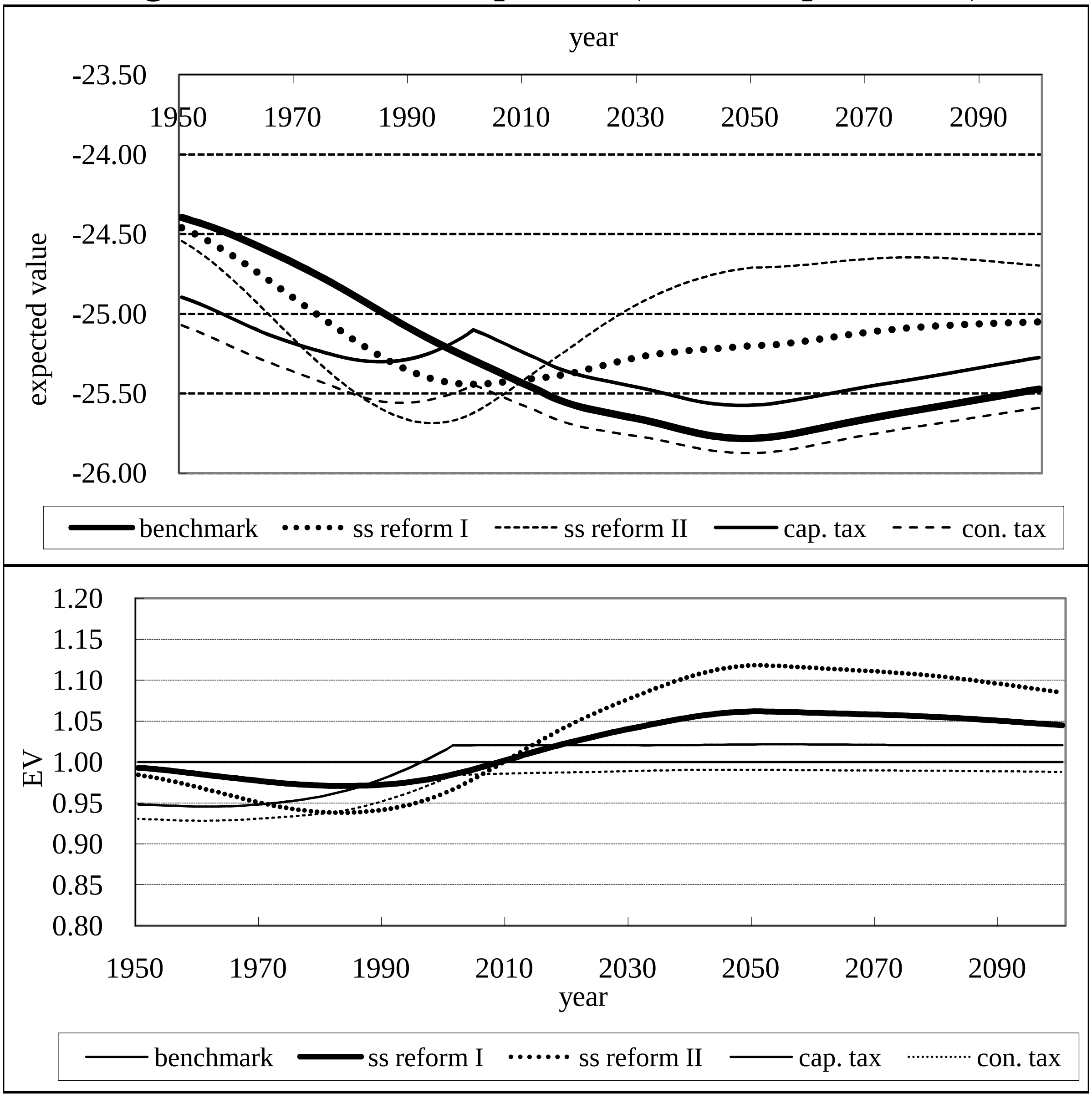


Figure 10: Earnings Profiles

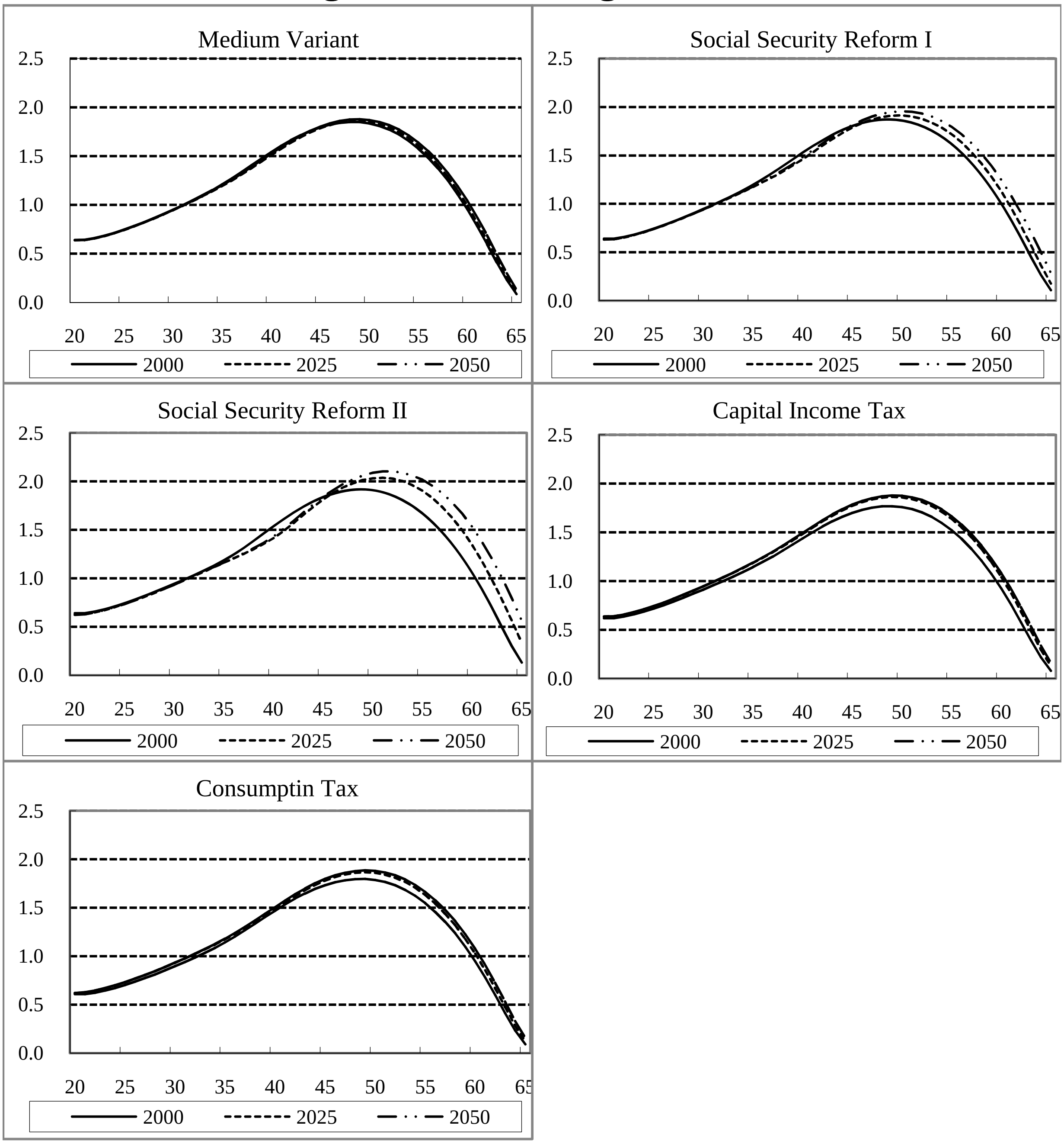


Figure 11: Asset Profiles

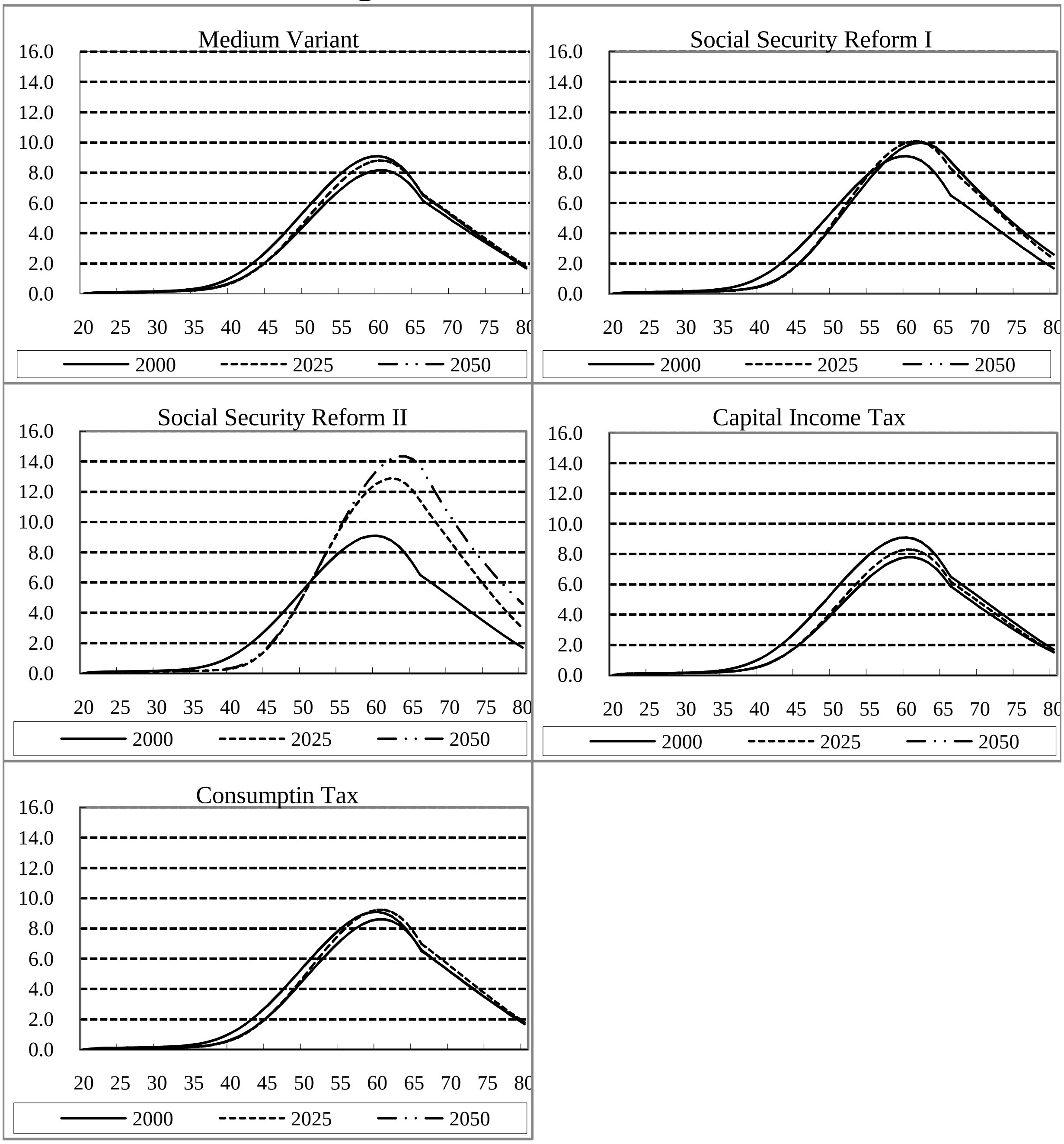


Figure 12: Consumption Profiles

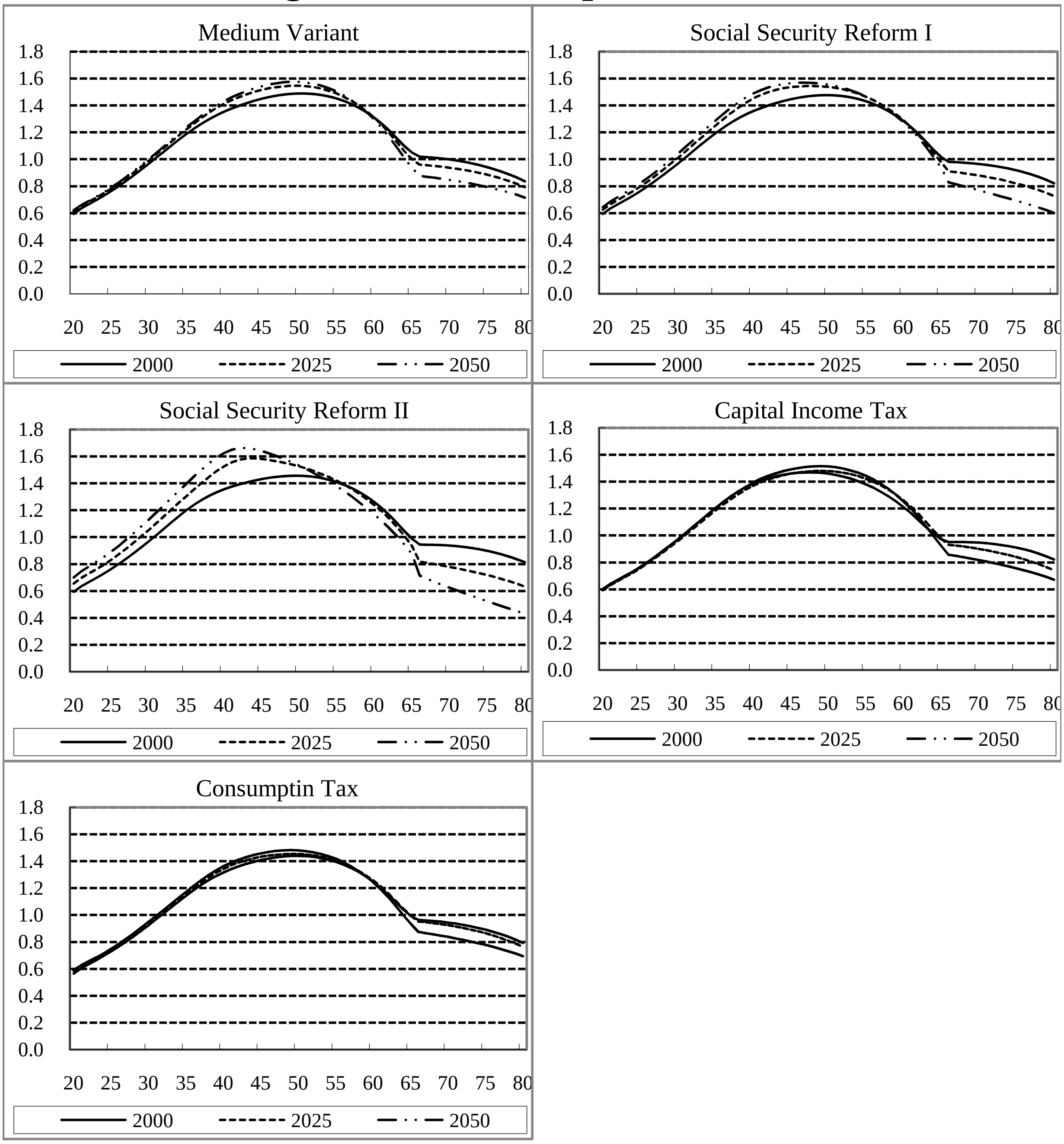


Figure 13: Transition Path (Low Variant)

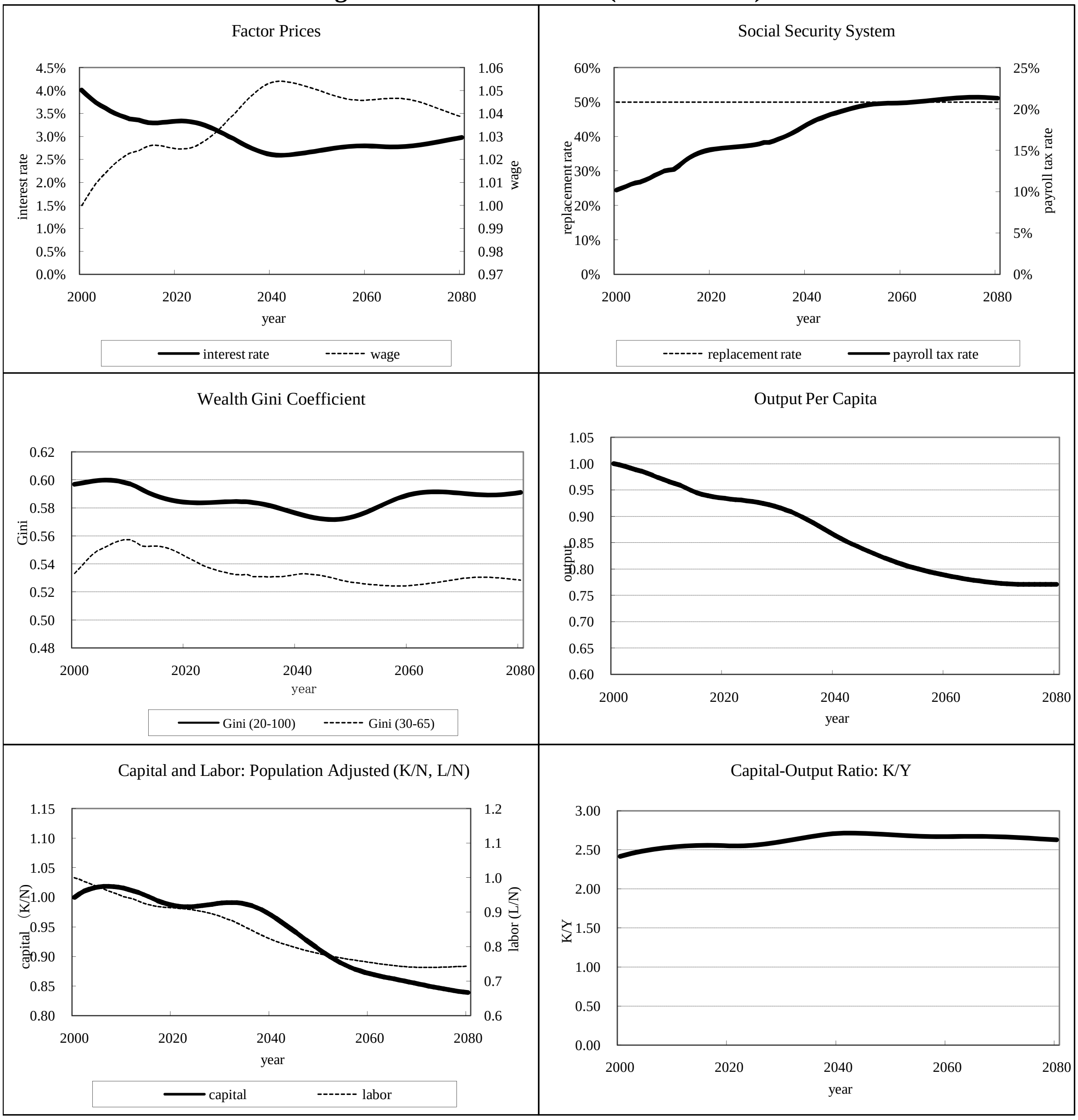


Figure 14: Transition Path (High Variant)

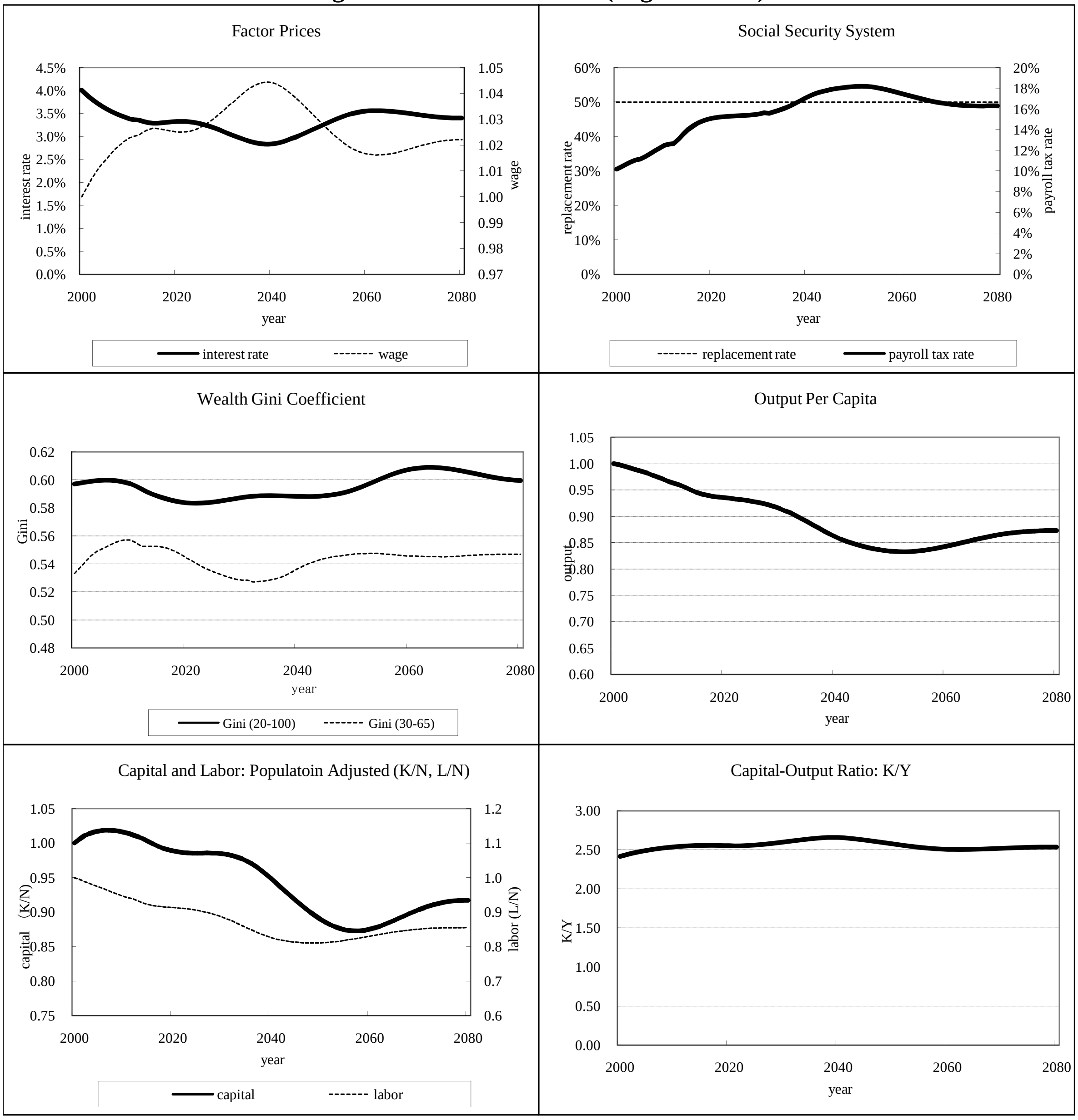


Figure 15: Transition Path with Government Fund